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MATCH for the Prosumer Smart Grid *The Algorithmics of Real-Time Power Balance*

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Abstract—*Prosumers* or proactive consumers are steadily on the rise in emerging Smart Grid systems. These consumers, apart from their traditonal role of using energy *from* the grid, also are actively involved in individually transferring stored energy from renewable sources such as wind and solar, *to* the grid. The large-scale integration of renewable generation in the emerging grid will re-define ways of meeting consumer energy demands, and more importantly drive greener and cost-effective utility operations. In this paper, we investigate the problem of matching consumer demand with the grid supply in real-time, and in the presence of renewables. We formulate this problem as a stochastic optimization problem and propose MATCH, a fast distributed real-time algorithm that accounts for the uncertainties in (i) renewable generation, (ii) the latter's transmission through the grid network, (iii) loads, and (iv) energy prices, and balances power in the Smart Grid at all times. MATCH is based on the Lyapunov stochastic optimization framework and scales to localities with a large number of networked renewable generation sources. We validate the efficacy of MATCH through experiments conducted using data modelled on proprietary data obtained from two public utilities. As part of the main results of this work, we show that (a) MATCH outputs unique approximate-optimal grid parameter configuration vectors in real-time that ensure perennial supply-demand balance in the grid at a minimum cost, and (b) mesh transmission network topologies lead to better MATCH outputs when compared to other existing transmission network topologies.

Keywords— prosumer, renewable energy, topology, power balance, optimization, MATCH

1 Introduction

A paradigm shift in the Smart Grid is the evolution of electricity customers from being passive consumers to electricity producer-consumers, i.e., *Prosumers* [1, 2, 3]. These consumers, apart from their traditional role of using energy from the grid, also are actively involved in individually transferring stored energy from renewable sources such as wind and solar, to the grid. The inclusion of Distributed Energy Resources (DER), including renewables and Energy Storage Systems (ESS) (such as the 10 kWh Powerwall Lithium Ion cell-based battery for residences recently introduced by Tesla Motors [4]) in the Smart Grid increases the complexity and variability in the grid and may significantly impact its reliability [5, 6, 7, 8, 9]. Out of the many connotations of reliability in the power grid, an important one is the ability of the grid to maintain *supply-demand balance* at all times. The importance of this reliability measure lies in an utility's ability to make grid operations more cost effective and environment friendly, i.e., failure to ensure supply-demand balance might result in the utility starting a new generation unit or buying expensive energy from external non-renewable and combustible generation sources that promote environmental pollution. Prosumers storing energy from renewable energy sources have the ability to ensure grid reliability by supplying demand deficits to the Smart Grid when a utility exhausts its conventional resources (e.g., coal), thereby alleviating the cost and pollutions issues that accompany buying non-renewable energy from external sources or starting a new generation unit, at times of demand deficit.

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1.1 Research Motivation

A major challenge to ensure reliability in the Smart Grid, i.e., maintaining power balance at all times, is the often intermittent, stochastic, and limited dispatchability nature of renewable generation in a dynamic prosumer energy network. Energy storage devices accompanying renewable generation sources is an environment friendly and cost effective way to tackle this challenge. In particular, the charging and discharging capability of storage can be exploited to shift energy across time, and the co-location of storage with renewable generators is often suggested [10]. Furthermore, many loads such as thermostatically controlled loads, electric vehicles (EVs), and other smart appliances, are time elastic and can be controlled via either curtailment or time shift. Thus, stored energy and elastic loads can jointly counter the fluctuations in renewable generation, and lead to a power balanced state in the Smart Grid at all times. However, their lies a final hurdle to effective power balance in the stochastic physical transmission network that transports the stored renewable energy to the grid. This network is prone to link losses and capacity variations. From a design perspective, power balancing between supply and demand in a power grid could be performed either separately or jointly on (i) the supply side, (ii) the demand side, and (iii) the storage side. *In this paper, we investigate the general problem of power balancing in a renewable-integrated dynamic power grid network with storage and elastic loads, considering the **joint** coordination of the supply side, demand side, and storage.* Given the potential need to perform energy management at a fine-grained time scale (potentially in seconds) in the emerging Smart Grid of the future, combined with its potentially large scale of prosumers, the need of the hour is to design efficient distributed and real-time algorithms for power balance for the dynamically networked Smart Grid. The need for a real-time algorithm is mainly motivated by the intermittence in renewable energy sources, which make accurate energy forecasts difficult to obtain, thereby rendering offline algorithms incapable of effective supply-demand balance. A distributed algorithm on the other hand eases the computational burden of a large Smart Grid sytem operator by offloading certain computational tasks to individual grid elements, thereby saving overall running time, when compared to a centralized algorithm.

1.2 Related Work

There are many recent works explicitly incorporating the grid system uncertainty into the energy management of the power grids. The authors of [11] and [12] consider power balance only via supply side management by assuming that all loads are uncontrollable, the authors of [13] study power balance only via demand side management by optimally scheduling non-interruptible and deferrable loads of individual users, and the authors of [14], [15], and [16] propose to employ energy storage to clear power imbalance. Some other works combine either supply side and demand side managements [17], or supply side and storage managements [18], or demand side and storage managements [19, 20, 21], in order to achieve power balance in the Smart Grid. Among existing works, [22] and [23] are mostly related to this work, in which all three types of energy management (i.e., supply, demand, and storage) are *jointly* considered for power balancing. However, the algorithm in [22] is distributed in nature but designed for *offline* use such as in day-ahead scheduling, and therefore cannot be implemented in real time. In [23], a real-time algorithm for power balance is proposed but the algorithm is executed centrally through a system operator. *In addition, none of the works from [11]-[23] consider the stochastic network effects of renewable energy transmission, and the role of the transmission network topology in effectively achieving the power balance task.* As a matter of fact, for the emerging dynamic prosumer-based grid, it is an important challenge to design and maintain reliable transmission network topologies that promote the achieving of real-time supply-demand balance with a high probability at low costs. To the best of our knowledge, this is the first work that addresses all the drawbacks of the above mentioned works and proposes a distributed real-time algorithm for power balance in the dynamic Smart Grid.

In relation to the salient methodologies used in this paper to solve the power balance problem, (i) the use of Lyapunov optimization framework for stochastic optimization is not new and has been adopted by some of the above mentioned works [22][23], however, in this paper we use it to solve the power balance at problem by explicity modeling the grid network topology, and (ii) the use of *Alternating Direction Method of Multipliers (ADMM)* is not new either and we use its power to achieve faster convergence for the topology-driven power balance problem compared to the convergence rates obtained via seminal subgradient-based algorithms [24]. In a very recent work [25], the author proposes a general framework for joint detection-learning-control algorithm design, and develop the receding learning-aided control (RLC) algorithm for potentially non-stationary dynamic systems such as the Smart Grid. RLC is an *online* algorithm that requires zero a-priori statistical knowledge and aids in the fast convergence of reaching optimal solutions to resource allocation problems at minimum cost. RLC quickly detects changes in system dynamic statistics via receding sampling, and efficiently incorporates learned system information into network control via dual learning and drift-augmentation. Even though RLC works well in non-stationary environments of zero system statistical knowledge, for our given data sets, we found it more fitting to adopt a real-time solution to the power balance problem with the assumption of system stationarity. However, we emphsize that the methodology in [25] can also be used to solve our problem.

1.3 Research Contributions

We make the following research contributions in this paper.

- We propose our dynamic Smart Grid system model, and formulate the real-time power balance problem as a constrained stochastic network optimization module. Our proposed model of the Smart Grid paints a realistic picture of the emerging grid, and captures the relevant system uncertainties prevalent in a prosumer-based energy network (See Section 2).
- Despite capturing the realistic system constraints in a dynamic Smart Grid, we show that our constrained power balance optimization task does not allow for the design of real-time algorithms that will find the optimal solution, i.e., system parameters that ensure power balance in the grid at minimum cost. To alleviate this issue, we reduce the problem formulation in Section 2 to an equivalent formulation on which real-time algorithms could be developed to reach *approximately optimal* solutions. In this regard, we propose a real-time algorithm, MATCH, to solve our modified stochastic network optimization problem for the optimal system parameters. MATCH is based on the theory of Lyapunov optimization. We characterize the performance (cost to ensure balance), gap, due to MATCH away from the optimum, and show that the algorithm is asymptotically optimal as the renewable energy storage capacity increases for prosumers, and the ramping constraint of the non-renewable generation unit loosens (See Section 3).
- We design a distributed version of MATCH that scales with the number of prosumer-contributed energy storage devices in a power network, and enjoys a fast convergence rate (See Section 4).
- We validate the efficacy of MATCH through experiments conducted using data modeled on proprietary data obtained from two public utilities, on various practical transmission network topologies. We show that mesh transmission network topologies result in the best system performance using MATCH, compared to point-to-point and parallel edge topologies (See Section 5).

2 Problem Setup

In this section, we propose our system model, and follow it up with formulating the power-balance optimization task

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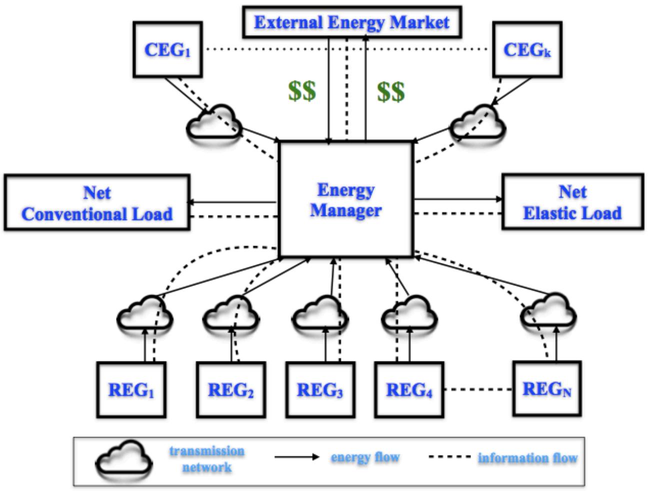

Fig. 1: A Representative Prosumer Smart Grid Architecture

for our model.

2.1 System Model

Here, we first provide a comprehensive overview of our power grid setting, which is followed by a description of the primary elements of the power grid. A diagrammatic representation of the comprehensive overview is shown in Figure 1.

2.1.1 Comprehensive Overview

We consider a power grid setting consisting of one conventional energy generator (CEG) (e.g., nuclear, coal-fired, or gas-fired generator) and N renwewable energy generators (REGs) (e.g., wind or solar generators). We note here that our work easily extends to the case of multiple CEGs. Each REG is co-located with an on-site energy storage unit. The power grid consists of an energy manager (EM) that manages all the possible energy sources, both renewable and non-renewable, balances the power across the grid (by satisfying load demands), and in case of energy shortage connects itself to external energy markets to buy energy. The CEG and REGs are connected to the EM via an overlay *star network topology* through which both, energy as well as information flows. Information flows are possible due to the Advanced Metering Infrastructure (AMI) available in the Smart Grid. The energy flows are unidirectional from the CEG and REGs to the EM, where information flows are bidirectional between the EM and the CEG/REGs. We assume that there is no connectivity between the REGs, or between the CEG and any REG. We also assume that the system operates in finite discrete time with time slot $t \epsilon \{0, 1, 2, \cdot, \cdot, T\}$. As a result, we will work with energy units instead of power units, throughput the paper, i.e., we only deal with the energy generated within a time slot of the form $[t-1, t]$, rather than the instantaneous power at any point in the interval.

2.1.2 Description of Primary Grid Elements

We have the following main elements characterizing the emerging prosumer-driven Smart Grid.

The Transmission Network - We assume an overlay star topology between the REGs, CEG, and the EM. A transmission network is dynamic and prone to losses, and we model the losses to grid energy transfers via random variable coefficients $\Phi^r_{i,t} \in [0, 1]$, and $\Phi^c_t \in [0, 1]$ denoting the *transmission efficiency* from renewable energy and conventional energy sources (See below), respectively, to the energy manager (EM) at various discrete time instants. We note that each overlay edge in the star topology could be comprised of a *single* physical link, *parallel physical links* representing multiple paths through which energy can be transferred from an energy source to the grid (EM), or a *network* of physical links between the generating source, and the EM sink. In this regard, $\Phi^r_{i,t}$ and Φ^c_t represent the efficiency coefficients in the physical sense, i.e., taking the physical link topology of an overlay edge into account. The transmission efficiency parameters for a given (source, sink) pair, for a general physical link topology with stochastic/dynamic links can be computed using the seminal theory proposed in [26] in conjunction with Monte Carlo methods.

Conventional Energy Generator (CEG) - A CEG generates energy from conventional energy sources like coal and nuclear. The energy output of the CEG is controllable. We denote CG_t to be the random variable describing the random net energy output of the CEG during time slot t, satisfying $0 \le CG_t \le cg_{\max}$, where $cg_{\max}$ is the maximum amount of the energy output. Due to the operational limitations of the CEG, the change of the outputs in two consecutive time slots is bounded instead of being arbitrarily large. This practical limitation is typically reflected by a *ramping constraint* on the CEG outputs [27]. Assuming that the ramp-up and ramp-down constraints are identical, we express the overall ramping constraint as

$$|CG_t - CG_{t-1}| \le r \cdot cg_{max},$$

Here, $CG_t = \Phi^c_t CEG_t$, where CEG_t is the energy generated by a CEG at time slot t without any transmission losses. The ramping coefficient, r, $0 \le r \le 1$, indicates the tightness of the ramping requirement. In particular, for $r = 0$, the CEG produces a fixed output over time, while for $r = 1$, the ramping requirement becomes non-effective. Furthermore, we denote the generation cost function of the CEG by $CCG()$.

Renewable Energy Generators (REGs) - A REG generates energy from renewable sources such as wind and solar. For a given REG i, we denote the amount of the renewable generation during time slot t by the random variable $RG_{i,t} \in [0, rg_{i,\max}]$. The rationality behind $RG_{i,t}$ being a random variable is the stochastic nature of the renewable sources. We assume that each REG is co-located with one on-site energy storage unit capable of charging and discharging. We denote the charging or discharging energy amount of the i-th storage unit during time slot t by the random variable $X_{i,t}$, with $X_{i,t} > 0 (resp.\, X_{i,t} < 0)$ indicating charging (resp. discharging). Because of the battery design and hardware constraints, the value of $X_{i,t}$ is bounded by the interval $[x_{i,\min}, x_{i,\max}]$ containing the maximum charging and discharging amounts. Let $ES_{i,t}$ be random variable denoting the energy state of the i-th colocated storage unit during time slot t, where $ES_{i,t} \in [es_{i,\min}, es_{i,\max}]$. We have the following relation describing the evolution of $ES_{i,t}$.

$$ES_{i,t+1} = ES_{i,t} + X_{i,t}.$$

During every time slot, an REG supplies energy to the energy manager. We denote the amount of the contributed energy by the i-th REG (including transmission losses) during time slot t by the random variable $R_{i,t}$. Since the energy flows of the REG should be balanced, we have

$$R_{i,t} = \Phi^r_{i,t}(RG_{i,t} - \eta X_{i,t}),\ R_{i,t} \geq 0, \eta \in [0,1],$$

where η is the charging/discharging efficiency of the storage unit. In particular, if $X_{i,t} > 0$ (charging state), the contributed energy $R_{i,t}$ directly comes from the renewable generation; if $X_{i,t} < 0$ (discharging state), $R_{i,t}$ comes from both the renewable generation, as well as storage unit.

Grid Loads - We model two types of loads: (a) conventional loads (CLs), and (b) elastic loads (ELs). Conventional loads represent critical energy demands such as lighting, that must be satisfied once requested. The elastic loads here represent some time-controllable energy requests that can be partly curtailed or shifted over time, if the energy provision cost is high. At time slot t, we denote the amount of the total requested conventional load by the *random variable* $L_{c,t}$, where $L_{c,t} \in [l_{c,\min}, l_{c,\max}]$. Similarly, we denote the amount of the total requested elastic load by $L_{e,t}$ where $L_{e,t} \in [l_{e,\min}, l_{e,\max}]$. The amounts $L_{c,t}$ and $L_{e,t}$ are generated by consumers based on their own needs. We denote the amount of the total load at time t to be $L_{tot,t}$, which satisfies $L_{tot,t} \leq L_{c,t} + L_{e,t}$. The control of ELs need to meet certain quality-of-service (QoS) requirement. In this paper, we impose an upper bound on the portion of unsatisfied elastic loads. Formally, we introduce the following constraint.

$$\frac{1}{T}\sum_{t=0}^{T-1} \mathrm{E}\left[\frac{l_{c,t} + l_{e,t} - l_{tot,t}}{l_{e,t}}\right] \leq \alpha,$$

for a given T. Here, $\alpha \in [0,1]$, is generally less than 0.05 in practice to indicate a tight QoS requirement.

Energy Trading - It is quite possible in the process of supply-demand balance that internal energy resources within the grid get exhausted. In such situations, the energy manager can resort to external energy markets if needed. For example, the energy manager can buy energy from the external energy markets in the case of energy deficit, or sell energy to the markets in the case of energy surplus. At time slot t, we denote the unit prices of the external energy markets for buying and selling energy by random variables $P_{buy,t} \in [p_{buy,\min}, p_{buy,\max}]$, and $P_{sell,t} \in [p_{sell,\min}, p_{sell,\max}]$, respectively. We typically keep the prices $P_{buy,t}$ and $P_{sell,t}$ random to model unexpected market behaviors. To avoid energy arbitrage, the buying price is assumed to be strictly greater than the selling price, i.e., $P_{buy,t} > P_{sell,t}$. Let $e_{buy,t}$ and $e_{sell,t}$ be the energy amount bought and sold by the energy manager from/to external sources.Then, we have the following relation to ensure balance in the smart grid.

$$CG_t + e_{buy,t} + \sum_{i=1}^{N} R_{i,t} = e_{sell,t} + L_{tot,t}.$$

We note that in the absence of an energy trading environment, the supply-demand balance condition at a particular time slot t would be an inequality of the form

$$CG_t + \sum_{i=1}^{N} R_{i,t} \geq L_{tot,t},$$

which might be harder to satisfy in practice, both, in a single time-slot, as well as across multiple time slots. In this work we consider the presence of an energy trading environment, as a practically viable design choice.

2.2 Optimization Problem Formulation

In this section, we formulate a stochastic optimization problem based on the above described system model. In an intuitive sense, it 'costs' the energy manager to maintain power-balance in the grid at real-time. In this regard, our main goal in this section is (i) to propose our cost metric for the energy manager, and (ii) formulate a constrained optimization problem whose solution minimizes this cost of ensuring supply-demand balance in the grid at all times.

We define the cost metric for the energy manager (EM) in our work as a random variable CM_t as follows.

$$CM_t = CCG(CEG_t) + \sum_{i=1}^{N} DG_i(X_{i,t}) + P_{buy,t}e_{buy,t} - P_{sell,t}e_{sell,t}.$$

The first term of the metric is the cost to the grid to produce energy from conventional energy sources at time slot t. The second term is the total degradation cost of the renewable energy battery storage for charging or discharging amount $X_{i,t}$, during time slot t, across all the batteries [28]. The third and fourth terms represent the cost and revenue to the grid respectively for buying and selling energy during time slot t. We now provide the formulation of our stochastic optimization problem, which we term as ST-OPT, and that follows from the system model in Section 2.1.

$$\textbf{ST-OPT:}\ \arg \min_{\{\overrightarrow{R_t}, \overrightarrow{X_t}, L_{tot,t}, CEG_t, e_{buy,t}, e_{sell,t}\}} \frac{1}{T}\sum_{t=0}^{T-1} \mathrm{E}[CM_t],$$

where

$$\overrightarrow{R_t} = \{R_{1,t}, R_{2,t}, ..., R_{N,t}\};\ \overrightarrow{X_t} = \{X_{1,t}, X_{2,t}, ..., X_{N,t}\}$$

subject to

$$L_{tot,t} \leq L_{c,t} + L_{e,t},\ \forall t. \tag{1}$$

$$L_{c,t} \in [l_{c,\min}, l_{c,\max}];\ L_{e,t} \in [l_{e,\min}, l_{e,\max}],\ \forall t. \tag{2}$$

$$\frac{1}{T}\sum_{t=0}^{T-1} \mathrm{E}\left[\frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}}\right] \leq \alpha. \tag{3}$$

$$X_{i,t} \in [x_{i,\min}, x_{i,\max}],\ \forall i, t. \tag{4}$$

$$ES_{i,t+1} = ES_{i,t} + X_{i,t},\ \forall i, t. \tag{5}$$

$$R_{i,t} = \Phi^r_{i,t}(RG_{i,t} - \eta X_{i,t}),\ R_{i,t} \geq 0, \eta \in [0,1],\ \forall i, t, \tag{6}$$

where

$$\Phi^r_{i,t} \in [0,1], ES_{i,t} \in [es_{i,\min}, es_{i,\max}], RG_{i,t} \in [0, rg_{i,\max}]$$

.

$$R_{i,t} \in [0, r_{i,\max}],\ \forall i, t. \tag{7}$$

$$CG_{i,t} \in [0, cg_{\max}],\ \forall i, t. \tag{8}$$

$$CG_t = \Phi^c_t CEG_t, \forall t, \tag{9}$$

where $\Phi^c_t \in [0,1];\ CEG_t \in [0, \frac{cg_{max}}{\Phi^c_t}], \forall t.$

$$|CG_t - CG_{t-1}| \leq r \cdot cg_{\max},\ \forall t. \tag{10}$$

$$e_{buy,t} \geq 0,\ \forall t. \tag{11}$$

$$e_{sell,t} \geq 0, \forall t. \tag{12}$$

$$CG_t + e_{buy,t} + \sum_{i=1}^{N} R_{i,t} = e_{sell,t} + L_{tot,t}, \forall t. \tag{13}$$

Due to the stochastic nature of the problem variables, the objective function minimizes the *expected* cost of ensuring supply-demand balance averaged over the total number of discrete time slots. The functions CCG, and DG are assumed to be *convex and continously differentiable*, with $CCG'()$ and $DG'()$ being bounded by the intervals $[CCG'_{i,\min}, CCG'_{i,\max}]$ and $[DG'_{i,\min}, DG'_{i,\max}]$, respectively. Constraints (1) to (13) reflect the various constraints as per our system model described in Section 2.1. *Constraint (13) ensures power balance in the Smart Grid at all times.*

3 The Design of 'MATCH'

In this section, we design MATCH, a real-time supply-demand balancing algorithm for prosumer energy networks. The first part of the section derives the design of MATCH from a modified optimization framework of ST-OPT. The second part analyses MATCH for its effectiveness in ensuring real-time power balance at a minimum cost, and lays down corresponding long-term practical implications when optimal system operating points are achieved over every time slot. We provide non long-term practical implications of MATCH in Section 5.

3.1 The MATCH Algorithm

We will use the theory of Lyapunov optimization [29] to design a real-time algorithm that solves ST-OPT. The Lyapunov framework enables the design of real-time algorithms for optimization problems based upon complex dynamic systems, such as the emerging Smart Grid. However, given the formulation for ST-OPT, the Lyapunov optimization framework cannot be directly applied to it, simply because of the absence of certain non time-averaged constraints such as constraint (5) and (10). To alleviate this problem, we propose a modified relaxed formulation of ST-OPT, that is a real-time convex optimization formulation for ST-OPT and obeys the conditions required for us to apply the Lyapunov framework, but at the same time paves the path for a real-time algorithm that outputs a good *approximation* to the optimal solution to ST-OPT. We first define the relaxed formulation of ST-OPT, followed by the description of MATCH.

3.1.1 Modified ST-OPT

We define, M-(ST-OPT), the modified real-time convex formulation of ST-OPT as follows.

M-(ST-OPT): $\arg \min_{\{\overrightarrow{R_t}, \overrightarrow{X_t}, L_{tot,t}, CEG_t, e_{buy,t}, e_{sell,t}\}} OBJ_t,$

where

$$\begin{aligned} OBJ_t \;\; &= \;\; \left[\sum_{i=1}^{N} H \cdot DG_i(X_{i,t}) + (ES_{i,t} - \beta_i) X_{i,t} \right] \\ &+ \;\; H \cdot CCG(CEG_t) + H \cdot P_{buy,t} e_{buy,t} \\ &- \;\; H \cdot P_{sell,t} e_{sell,t} - \frac{J_t}{L_{e,t}} L_{tot,t}, \end{aligned}$$

where

$$H \in [0, H_{\max}];$$

$$H_{\max} = \min_{\forall i} \left\{ \frac{ES_{i,\max} + X_{i,\min} - X_{i,\max}}{P_{buy,\max} - P_{sell,\min} + DG'_{i,\max} - DG'_{i,\min}} \right\},$$

$$\beta_i = H(P_{buy,\max} + DG'_{i,\max}) - X_{i,\min},$$

and

$$J_t = \max\{J_{t-1} - \alpha, 0\} + \frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}}.$$

subject to

$$L_{tot,t} \leq L_{c,t} + L_{e,t}, \forall t. \tag{14}$$

$$L_{c,t} \in [l_{c,\min}, l_{c,\max}]; \; L_{e,t} \in [l_{e,\min}, l_{e,\max}], \forall t. \tag{15}$$

$$X_{i,t} \in [x_{i,\min}, x_{i,\max}], \forall i, t. \tag{16}$$

$$R_{i,t} = \Phi^r_{i,t}(RG_{i,t} - \eta X_{i,t}), \; R_{i,t} \geq 0, \eta \in [0,1], \forall i, t, \tag{17}$$

where $\Phi^r_{i,t} \in [0,1]$, and $RG_{i,t} \in [0, rg_{i,\max}]$.

$$R_{i,t} \in [0, r_{i,\max}], \forall i, t. \tag{18}$$

$$CG_{i,t} \in [0, cg_{\max}], \forall i, t. \tag{19}$$

$$CG_t = \Phi^c_t CEG_t, \forall t, \tag{20}$$

where $\Phi^c_t \in [0,1]$; $CEG_t \in [0, \frac{cg_{max}}{\Phi^c_t}], \forall t$.

$$|CG_t - CG_{t-1}| \leq r \cdot cg_{\max}, \forall t. \tag{21}$$

$$e_{buy,t} \geq 0, \forall t. \tag{22}$$

$$e_{sell,t} \geq 0, \forall t. \tag{23}$$

$$CG_t + e_{buy,t} + \sum_{i=1}^{N} R_{i,t} = e_{sell,t} + L_{tot,t}, \forall t. \tag{24}$$

The problem, M-(ST-OPT), is convex, suited for the use of the Lyapunov framework, and satisfies all the constraints imposed in ST-OPT. A striking difference of M-(ST-OPT) with ST-OPT is the absence of constraint (3) in ST-OPT in M-(ST-OPT). In order to meet constraint (3) in M-(ST-OPT), we introduce a virtual queue backlog J_t in line with the theory of Lyapunov optimization [29]. The virtual queue J_t accumulates the portion of elastic loads. Maintaining the stability of J_t is equivalent to satisfying (3) in ST-OPT [29]. Another striking difference of M-(ST-OPT) with ST-OPT is the absence of constraint (5) in ST-OPT in M-(ST-OPT). In this regard, the parameter β_i in M-(ST-OPT) guarantees the boundedness of $ES_{i,t}$ as expressed through constraint (5) in ST-OPT. Also note that to ensure the feasibility of CEG_t, constraint (10) in ST-OPT stays in M-(ST-OPT) as constraint (23), even though it is not time-averaged. The objective function, OBJ_t in M-(ST-OPT) is a drift-plus-cost function that is a linear combination of the system cost, drift related to the virtual queue J_t, and energy states. The goal of the optimization objective is to minimize this system cost. We show in Section 8 that the objective function is upper bounded. *For stochastic objective functions such as the one in M-(ST-OPT), an alternative primal-dual method makes decisions similar to drift-plus-penalty decisions, but uses a penalty defined by partial derivatives of the objective function.*

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Algorithm 1: MATCH - ensures real-time power balance

Input: $\overrightarrow{RG_t}, L_{c,t}, L_{e,t}, P_{buy,t}, P_{sell,t}$

Output: $\overrightarrow{R_t^{opt}}, \overrightarrow{X_t^{opt}}, L_{tot,t}^{opt}, CEG_t^{opt}, e_{buy,t}^{opt}, e_{sell,t}^{opt}$

Initialize $J_0 = 0$. EM does the following for every time slot t

while *time slots are not exhausted* **do**

Observe $\overrightarrow{RG_t}, L_{c,t}, L_{e,t}, P_{buy,t}, P_{sell,t}, \overrightarrow{ES_t}, J_t$.

Solve M-(ST-OPT) for $\overrightarrow{R_t^{opt}}, \overrightarrow{X_t^{opt}}, L_{tot,t}^{opt}, CEG_t^{opt}, e_{buy,t}^{opt}, e_{sell,t}^{opt}$.

Use the solution to M-(ST-OPT) to update $\overrightarrow{ES_t}, J_t$ based on the following:

$$J_t = \max\{J_{t-1} - \alpha, 0\} + \frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}},$$

and

$$ES_{i,t+1} = ES_{i,t} + X_{i,t}, \forall i, t.$$

3.1.2 Description of MATCH

The real-time MATCH algorithm is described in **Algorithm 1.** MATCH needs to be run by the energy manager at every time slot.

3.2 Algorithm Analysis

In this section, we analyse the performance of MATCH, and compare the optimal time-averaged solution of M-(ST-OPT) usign MATCH, denoted by $CM^{mst-opt}$, with the optimal time-averaged solution of ST-OPT, denoted as CM^{st-opt}. The performance metric is the system cost to ensure supply-demand balance in the grid at all times. In this work we consider the time-averaged system cost over all time slots. We have the following theorem in this regard, the proof of which is in Section 8.

Theorem 1. *Given that* $\overrightarrow{RG_t}, L_{c,t}, L_{e,t}, P_{buy,t}, P_{sell,t}$ *are i.i.d's, we have*

1) $\overrightarrow{R_t^{opt}}, \overrightarrow{X_t^{opt}}, L_{tot,t}^{opt}, CEG_t^{opt}, e_{buy,t}^{opt}, e_{sell,t}^{opt}$ *is a feasible solution to ST-OPT.*
2) $CM^{mst-opt}(r, H) - CM^{st-opt}(r) \leq K$, *where*
$$K = (1-r)\frac{cg_{max}}{\Phi_t^c}\max\{p_{buy,\max}, CCG'_{\max}\} + \frac{B}{H}.$$
Here,
$$CM^{mst-opt} = \sup \frac{1}{T}\sum_{t=1}^{T} CM_t^{mst-opt};$$
is the optimal solution produced by MATCH for M-(ST-OPT), and
$$CM^{st-opt} = \sup \frac{1}{T}\sum_{t=1}^{T} CM_t^{st-opt},$$
and
$$B = \frac{1}{2}\left\{(1+\alpha^2) + \sum_{i=1}^{N}\max\{x_{i,\min}^2, x_{i,\max}^2\}\right\}$$
3) $CM^{st-opt}(r) \geq CM^{mst-opt}(1, H) - \frac{B}{H}$.
4) $ES_{i,t} \in [0, es_{i,top}]$, *where* $es_{i,top}$ *equals*
$$H(p_{buy,\max} - p_{sell,\min} + DG'_{i,\max} - DG'_{i,\min}) - x_{i,\min} + x_{i,\max}.$$
5) $J_t \in [0, H \cdot p_{buy,\max} l_{e,\max} + 1]$; $e_{buy,t}^{opt} \cdot e_{sell,t}^{opt} = 0$. *Here,* $e_{buy,t}^{opt}$ *and* $e_{sell,t}^{opt}$ *are outputs of MATCH.*

Theorem Implications: The important practical implications of Theorem 1 are (i) The performance gap between the optimal solution to ST-OPT and M-(ST-OPT), i.e., the difference between the time averaged costs of ST-OPT and M-ST-OPT is bounded from above by K, which intuitively *asymptotically* reaches zero with increasing H (the capacity of a storage unit), and the loosening of the ramping parameter, r (following from Theorem 1.2 and 1.3), (ii) the results will hold true even for *non* i.i.d. $\overrightarrow{RG_t}$, $L_{c,t}$, $L_{e,t}$, $P_{buy,t}$, $P_{sell,t}$, and extends to the case when these random variables evolve based on a *finite state*, *irreducible*, and *aperiodic* Markov chain (follows from the applicability of the Lyapunov optimization framework to non i.i.d. settings), (iii) the maximum value of the energy state (the ES variable) for each storage device (a) increases linearly with H, (b) is larger if the energy prices are more volatile or the marginal degradation cost increases fast, and (c) is minimum if the prices for buying and selling energy are equal and constant, and the degradation cost is zero (follows from Theorem 1.4), and (iv) the queue backlog, i.e., the portion of unsatisfiable elastic loads, is upper bounded and the energy manager (EM) does not simultaneously buy or sell energy (follows from Theorem 1.5).

4 DESIGNING DISTRIBUTED (D) MATCH

Algorithm MATCH can be used by the energy manager at every time slot to ensure supply-demand balance in the power grid. However, MATCH is centralized in nature. In reality, we could be looking at potentially a million renewable energy sources in a large metropolitan locality. In such a setting, the REGs may not be willing to give direct control of storage or offer private information to the centralized EM regarding the same. In addition, the computational complexity of centralized control would grow quickly as the number of REGs increase. In this section, we propose D-MATCH, a distributed version of the MATCH algorithm for solving M-(ST-OPT). We first lay down the theory behind our distributed algorithm, which is followed by the description of the practical implementation aspects of D-MATCH.

4.1 The Theory Behind D-MATCH

Our intuition here is to formulate M-(ST-OPT) in a manner so as to be able to use the Alternating Direction Method of Multipliers (ADMM) approach proposed in [30] for the design of a distributed version of MATCH through an iterative process. We denote this new formulation, D-(ST-OPT), as follows.

$$\textbf{D-(ST-OPT)} \qquad \arg\min_{\overrightarrow{\gamma_t}, \overrightarrow{\delta_t}} OBJ_t,$$

where

$$OBJ_t = \sum_{i=1}^{N+4}[F_{i,t}(\alpha_{i,t}) + \mathbf{1}(\gamma_{i,t} \in \Gamma_{i,t})] + \mathbf{1}\left(\sum_{i=1}^{N+4}\delta_{i,t} = \sum_{i=1}^{N}\Phi^r_{i,t}RG_{i,t}\right),$$

subject to

$$\overrightarrow{\gamma_t} - \overrightarrow{\delta_t}, \forall t. \tag{25}$$

Here, $\mathbf{1}(\cdot)$ is an indicator function that equals 0 if the enclosed event is true and infinity otherwise. $\overrightarrow{\gamma_t} = [\gamma_{1,t}....,\gamma_{N+4,t}]$ is the vector of random variables related to the M-(ST-OPT) by the relations $\gamma_{i,t} = X_{i,t}$ for $1 \le i \le N$, $\gamma_{N+1,t} = L_{tot,t}$, $\gamma_{N+2,t} = -CEG_t$, $\gamma_{N+3,t} = -e_{buy,t}$, and $\gamma_{N+4,t} = e_{sell,t}$. $\Gamma_{i,t}$ is the constraint set associated with γ_i, and function $F_{i,t}(\gamma_{i,t}$ is part of the objective if M-(ST-OPT) and associated with the optimization variable in M-(ST-OPT) that is equivalent to $\gamma_{i,t}$. The expression inside the indicator variable of the second half of OBJ_t denotes the supply-demand balancing requirement during time slot t. Note that $\overrightarrow{\delta}$ is an auxilliary variable and copy of $\overrightarrow{\gamma}$.

In order to solve D-(ST-OPT) we first introduce the dual random variable vector $\overrightarrow{d_t} = [d_{1,t}, d_{2,t}, ..., d_{N+4,t}]$. Using the general ADMM approach, D-MATCH uses an iterative scheme where, at the $k+1$-th iteration, the variables $\gamma_{i,t}$, $\delta_{i,t}$, and $d_{i,t}$ are updated as follows:

$$\gamma^{k+1}_{i,t} = \underset{\gamma_{i,t}}{\operatorname{argmin}}\left\{F_{i,t}(\gamma_{i,t}) + \frac{\rho}{2}(\gamma_{i,t} - \delta^k_{i,t} + \frac{d^k_i}{\rho})^2 | \gamma_{i,t} \in \Gamma_{i,t}\right\}, \forall i, t, \tag{26}$$

$$\overrightarrow{\delta^{k+1}_t} = \underset{\overrightarrow{\delta_t}}{\operatorname{argmin}}\left\{\sum_{i=1}^{N+4}(\delta_{i,t} - \frac{d^k_{i,t}}{\rho} - \gamma_{i,t})^2 | \sum_{i=1}^{N+4}\delta_{i,t} = \sum_{i=1}^{N}\Phi^r_{i,t}RG_{i,t}\right\}, \tag{27}$$

and

$$d^{k+1}_{i,t} = d^k_{i,t} + \rho(\gamma^{k+1}_{i,t} - \delta^{k+1}_{i,t}), \forall i, t. \tag{28}$$

Here, ρ is a positive penalty parameter, which needs to be adjusted for good convergence performance [30]. We now define AVG($\gamma^k_{i,t}$) per time slot t to equal $\frac{1}{N+4}\sum_{i=1}^{N+4}\gamma^k_{i,t}$. We also define AVG($d^k_{i,t}$) per time slot t to equal $\frac{1}{N+4}\sum_{i=1}^{N+4}d^k_{i,t}$. Solving the optimization problem in (31) then yields

$$\delta^{k+1}_{i,t} = \frac{d^k_{i,t}}{\rho} + \gamma^{k+1}_{i,t} - \frac{AVG(d^k_{i,t})}{\rho} - AVG(\gamma^{k+1}_{i,t}) + \frac{\sum_{i=1}^{N}\Phi^r_{i,t}RG_{i,t}}{N+4}. \tag{29}$$

Replacing the right hand side of (33) for $\delta^{k+1}_{i,t}$ in (31), we get

$$d^{k+1}_{i,t} = AVG(d^k_{i,t}) + \rho\left(AVG(\gamma^{k+1}_{i,t}) - \frac{\sum_{i=1}^{N}\Phi^r_{i,t}RG_{i,t}}{N+4}\right), \forall i, t. \tag{30}$$

Equation (34) implies that the dual variables $d^{k+1}_{i,t}$ are identical for all i for each iteration in time slot t. Thus, we rewrite (34) as

$$d^{k+1}_{t} = AVG(d^k_{t}) + \rho\left(AVG(\gamma^{k+1}_{i,t}) - \frac{\sum_{i=1}^{N}\Phi^r_{i,t}RG_{i,t}}{N+4}\right), \forall t. \tag{31}$$

We again replace the right hand side of (33) for δ_i, t^k in (30), and using the fact that (i) d^k_i are identical for all i, and (ii) $\overrightarrow{\delta_t}$ is not used for the $\overrightarrow{\gamma_t}$ or dual variable updates, yields a modified (30) as follows.

$$\gamma^{k+1}_{i,t} = \underset{\gamma_{i,t}}{\operatorname{argmin}}\left\{F_{i,t}(\gamma_{i,t}) + \frac{\rho}{2}(\gamma_{i,t} - \psi^k_{i,t})^2 | \gamma_{i,t} \in \Gamma_{i,t}\right\}, \forall i, t, \tag{32}$$

where

$$\psi^k_{i,t} = \gamma^k_{i,t} - AVG(\gamma^{k+1}_{i,t}) - \frac{d^k_t}{\rho} + \frac{\sum_{i=1}^{N}\Phi^r_{i,t}RG_{i,t}}{N+4}, \forall i, t,$$

Based on the theory in [31], the above updates lead to a worst case convergence rate of $O(\frac{1}{k})$ for D-MATCH, which is much faster than the convergence rates obtained by the use of the seminal subgradient-based algorithm in [24].

4.2 Implementing D-MATCH in Practice

The optimization problem in (30) can be solved separately at each REG, $i \in \{1, 2, .., N\}$, and at the EM for $i \in \{N+1, .., N+4\}$, whereas the updating task in (31) can be computed solely by the EM. At the initial iteration, each REG needs to send the amount of its renewable generation $\Phi^r_{i,t}RG_{i,t}$ to the EM. Following this, the EM emits a signal $\psi^k_{i,t}$ to each REG for its computation, and the REG solves (30) and sends the solution $\gamma^{k+1}_{i,t}$ to the EM. The EM finally integrates the optimal solutions from each REG. We see that the REGs do not have to release any other private information to the EM, and the required information exchange is limited. The optimization problems in (30) are all strictly convex and admit a unique (and sometimes closed-form) solution. Furthermore, in (31), only one dual variable is finally required to be updated. This is because the transformation of M-(ST-OPT) and the introduction of the new optimization variables $\overrightarrow{\gamma_t}$ permit all dual variables to share the same form of update, hence effectively reducing the number of the actual dual updates, as well as simplifying the calculation.

5 Performance Evaluation

In this section, we describe our experimental setup, and follow it up with analysing the performance of MATCH and D-MATCH under our proposed experimental setting. Our setting proportionally reflects actual data sets from the Los Angeles Department of Water and Power (LADWP) and Sacramento Municipality Utility District (SMUD). *Due to the proprietary nature of LADWP and SMUD data, we are unable to report experimental results based upon actual data sets.*

5.1 Experimental Setup

We consider three types of physical network topologies for the transmission network between a CEG, REG source, and the EM sink: (i) a single point-to-point edge, (ii) a set of 10 parallel edges, and (iii) a randomly generated mesh network of 30 nodes. The individual reliability, i.e., transmission efficiency, of each edge is uniformly distributed in the interval [0,1]. The reliability of the entire transmission network from a given source to the EM is computed using the algorithm in [26]. We set the length of each time slot to be of 15 minutes, and run our experiments for 1000 time slots. The conventional load and elastic loads are uniformly distributed in the interval [5 kWh, 30 kWh] for each time slot. We vary α from 0.8 to 1.0, and choose the number of REGs to lie in the set {500, 1000, 10,000}. For each storage unit, the maximum discharging and charging amounts are set to 1.1 kWh, assuming the discharging and charging rate to be 6.6 kWh. We model the degradation cost function of storage i via the function of the form $DG_i(x_t) = 5x_t^2$. We assume $REG_{i,t}$ to be uniformly distributed in the range [0,

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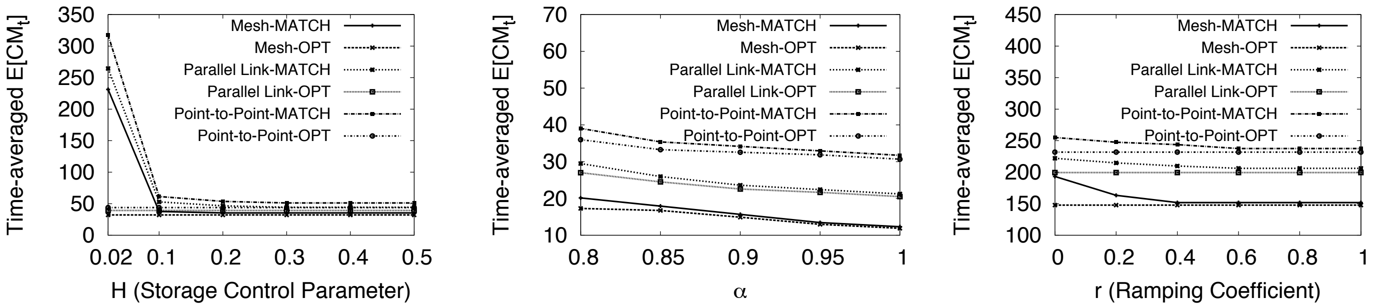

Fig. 2: Topology-Driven Performance w.r.t. (a) H (left), (b) α (middle left), (c) r (middle right), when #REGs = 500

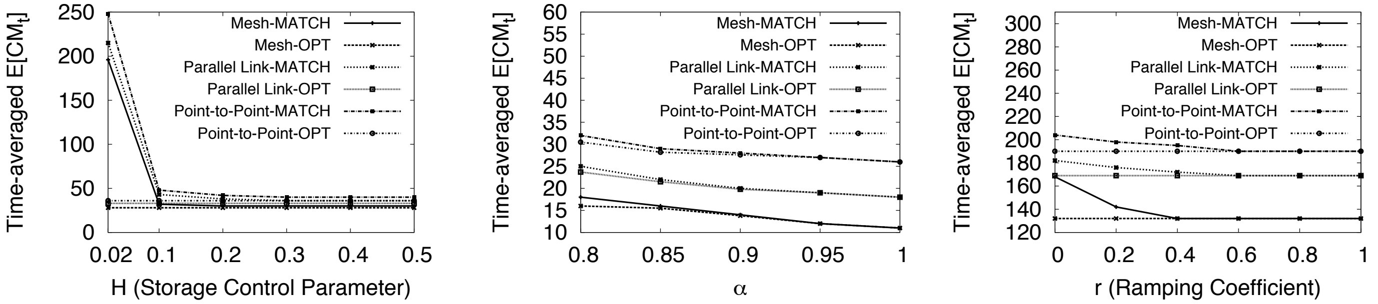

Fig. 3: Topology-Driven Performance w.r.t. (a) H (left), (b) α (middle left), (c) r (middle right), when #REGs = 1000

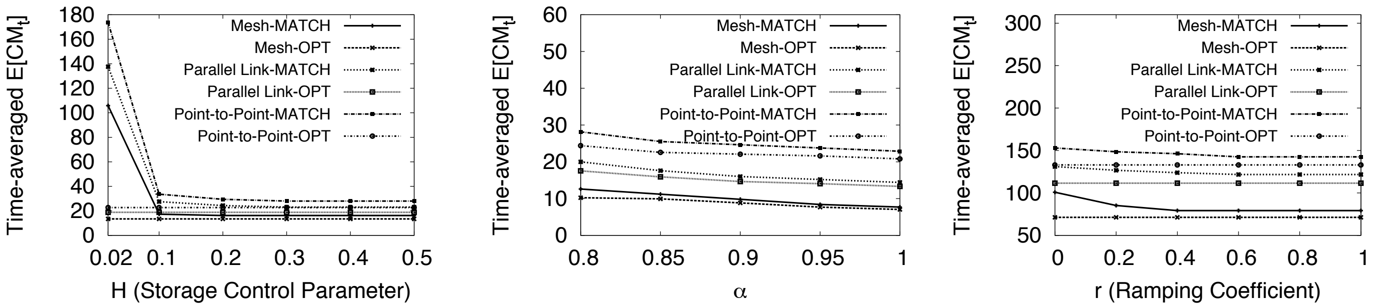

Fig. 4: Topology-Driven Performance w.r.t. (a) H (left), (b) α (middle left), (c) r (middle right), when #REGs = 10,000

1.1 kWh]. We design the generation cost function, $CCG(\cdot)$ for the CEG to be of the form $CCG(x_t) = 4x_t$, where $ceg_{\max}$ = 50 kWh. We fix the per unit energy buying price per time slot to be uniformly distributed in the interval [8, 10] cents per kWh. The per unit energy selling price per time slot is set to be uniformly distributed in the interval [4, 6] cents per kWh. For a random mesh topology, each plot point represents the average of 50 random topology instances. We use MATLAB and GNU plot to run our experiments and plot the results, respectively.

5.2 Results

We compare our experimental results of using MATCH with the lower bound of the optimal system cost obtained via Theorem 1.3 (we refer to this lower bound as 'optimal performance' in the text and OPT in the plots). We observe from Figure 2a. that (i) the system cost converges to a given value for low H values, (ii) even using low storage capacities (controlled via parameter H), one could achieve near optimal performance for all types of transmission network topologies under consideration. We observe in Figure 2b. that with increasing α, the system cost and its marginal decreases, and this is intuitive given that we have to satisfy a lesser load. In addition, the system cost from MATCH is close to the optimal performance for increasing α for all our network topologies, and equals it as $\alpha \rightarrow 1$. With respect to ramping requirements, we observe from Figure 2c. that the system cost monotonically decreases with increasing r. This is intuitive given the lesser use of bought expensive external energy, with increasing r. We also observe that for $r \geq 0.4$, the system cost stabilizes to a fixed value, this being due to the CEG supply being sufficient enough for ramping constraints to be relaxed any further. In addition, the system cost from MATCH approaches the optimal performance with increasing r. We study convergence of D-MATCH in Figure 5a. and observe that for various problem instances, the algorithm converges much faster than the seminal distributed subgradient approach in [24] (denoted by SA in the plot), for all topology types. The speed of convergence is measured through how fast the *average optimality gap* (the difference between the optimal solution to D-(ST-OPT)

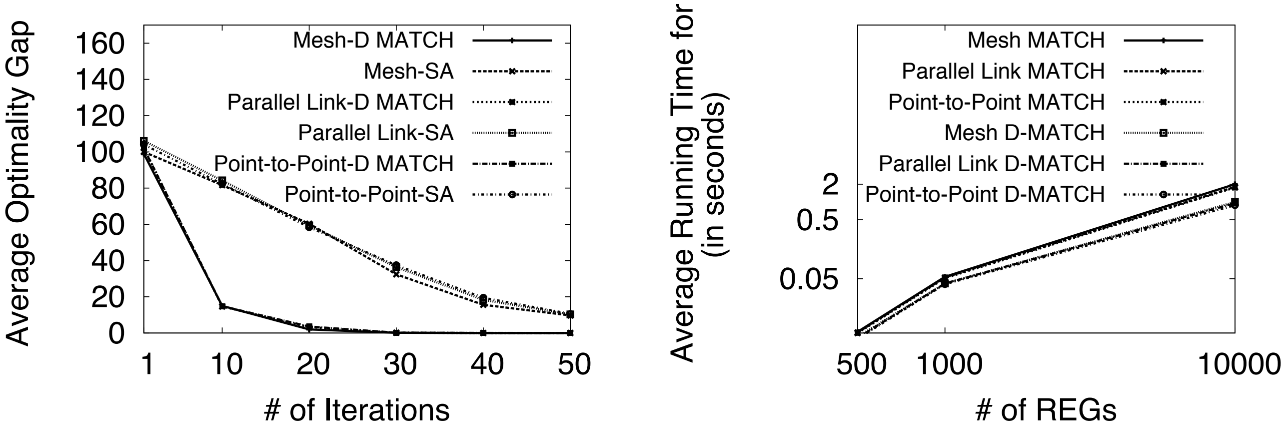

Fig. 5: Topology-Driven Performance w.r.t. (a) Average Optimality Gap (left), (b) Average Running Time (right)

and the output from D-MATCH/SA for D-(ST-OPT)) goes towards zero, where the average is taken across 50 random instantiations of a system with 10000 REGs. *A common takeaway message* ***(in turn a design advice for transmission network managers)*** *from our plots, apart from the efficacy of MATCH, is that under MATCH, mesh transmission network topologies from an energy source to the EM lead to the best system performance due to them opening up more disjoint energy flow paths from the source to the destination, thereby increasing power flow reliability.* In Figures 3 and 4, we plot Figures 2(a) - 2(c) with increasing count of the total number of REGs. Quite evidently, we observe that the system cost for the grid decreases with increasing REG count because there are more renewable resources to satisfy consumer demand in real-time, implying the grid incurring lesser costs in buying energy and generating more revenue through selling energy. Figure 5b. plots the actual average running time of MATCH and D-MATCH (without the information exchange times) on 50 random system instances consisting of 500, 1000, and 10,000 REGs respectively. We observe that the running time gap between MATCH and D-MATCH increases with increasing REG count, denoting the effectiveness of a distributed algorithm in significantly reducing the running time for large system instances.

6 Discussion

In this section, we briefly discuss a pathway to the realization in practice of smart power systems that integrate scalable, real-time, and distributed optimization components such as ours.

Traditionally, power grids have been designed as centrally controlled environments. Even with the introduction of smart meters, devices capable of being programmed remotely to sample and transmit data from households to the utility, this centralized approach has remained unchanged. However, the increased size of these emerging smart grids and the stress on the communication network pulling smart meter data every 15 minutes or less, are challenging this approach similarly with what the intercloud and the grid federation have done in the information technology (IT) domain. Communication networks used in smart grids (e.g., wireless, radio, Programmable Logic Controller (PLC)) cannot cope with the intense traffic required for real-time optimizations [32] such as the one we are proposing. The shift from a centralized control center to a more distributed one is motivated also by the introduction of DERs and renewables which can be independently owned and operated and shared through contracts with the utility. Owners of such energy sources may not be willing to release control or share sensitive data for security and privacy issues. This emerging heterogeneous and distributed smart grid requires a higher degree of voluntary collaboration between entities. In such a cooperative environment actors share limited amount of data with the central orchestrators (EM) and retain their independence in terms of energy generation and storage. Through contracts they can share their energy to handle global or local demand supply imbalances. This approach of pushing some of the computations towards the edges of the network, i.e., via fog computing, aims at reducing the communication bottleneck and the computational stress on the centralized controller.

In Section 4 we presented D-MATCH, a distributed algorithm for optimizing consumption during demand supply imbalances based energy coming from DERs and ESSs. The algorithms functionality depicted in Section 4.2 can be easily implemented in a distributed computing environment governed by contracts between the EM and the DER and ESSs owners. The architecture depicted in Figure 6 is similar to the MapReduce model where the smart grid is partitioned, data is independently processed and individual results are aggregated by the EM for orchestration. We should note that while the optimization itself is distributed, the control decision is centralized and enacted by the EM. Partitioning the smart grid can be done at various levels depending on its complexity and topology. Ideally, each CEG and REG would be independent but they can be aggregated to reduce the costs needed for installing the processing nodes. Smart meters would be ideal candidates for these processing nodes; however, while they can be used for controlling home area network appliances and can be programmed to sample and deliver data at various intervals, their ability to perform complex optimization problems is limited. Until such capability becomes a reality, compute nodes will need to be installed at every distributed point. These can range from ordinary computers to tablets linked to a home area network and will communicate with the EM following the flow depicted in Section 4.2. The distributed architecture minimizes the data to be transmitted over the network,

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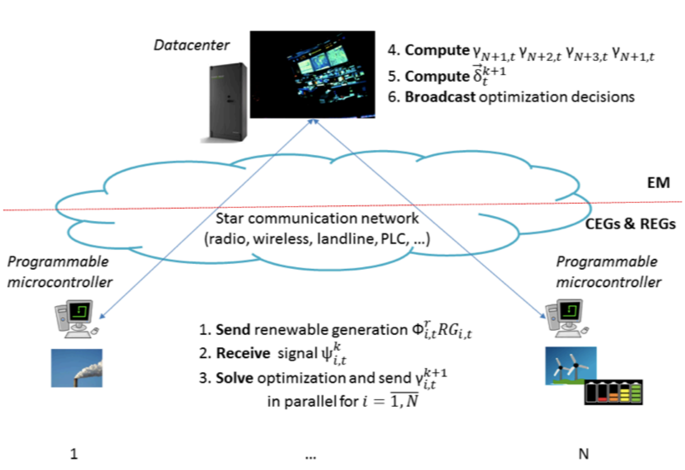

Fig. 6: Representative Architecture to Realize D-MATCH in Practice

reduces the stress on the EM, and maintains the privacy requirements of CEGs and REGs. To avoid saturating the transmission network our proposed algorithm can be executed only during unexpected events that can trigger load imbalances. Efficient data-driven prediction algorithms for modeling DER generation can be used outside these events to predict their impact on the network. During these unexpected events which cause demand spikes, the algorithm can balance the load by requesting renewable generation from CEGs and REGs and by running distributed optimizations. This avoids having to send renewable generation in real-time continuously to the EM.

7 Conclusion and Future Work

In this paper we addressed the problem of ensuring power balance in the prosumer-driven dynamic Smart Grid system at all times. We modeled the problem as a stochastic network optimization task to capture various uncertainties that characterize the sytem elements in practice. We designed MATCH, a fast, real-time, distributed algorithm based on the standard Lyapunov optimization framework, to solve the stochastic optimization task. We showed via theory and experiments that MATCH optimizes system cost to ensure real-time supply-demand balance, and also scales well to increasing number of renewable energy sources in the grid. We also showed that mesh transmission network topologies from an energy source to the grid lead to the best system performance, i.e., system cost, when using MATCH. As part of future work, we plan to deploy MATCH in field trials conducted by utility companies and study the effectiveness of our proposed algorithm in the day-to-day functioning of the Smart Grid.

8 Proof of Theorem 1

In this section, (i) we provide an upper bound of the objective function in problem M-(ST-OPT), and use it to (ii) prove Theorem 1.

Upper Bound of Objective Function. We define a vector $\Xi_t = [ES_{1,t}, ES_{2,t}, ..., ES_{N,t}, J_t]$, consisting of the energy states of all storage units, and the virtual queue backlog J_t, at time slot t. We define $L(\Xi_t)$ to be a Lyapunov function given by

$$L(\Xi_t) = \frac{1}{2}J_t^2 + \frac{1}{2}\sum_{i=1}^{N}(ES_{i,t} - \beta_i)^2.$$

We also define a conditional Lyapunov drift function given by

$$\Delta(\Xi_t) = \mathrm{E}[\mathrm{L}(\Xi_{t+1}) - \mathrm{L}(\Xi_t)|\Xi_t].$$

The drift-plus-cost function is given by $\Delta(\Xi_t) + H\mathrm{E}[\mathrm{CM_t}|\Xi_t]$, which is a weighted sum of $\Delta(\Xi_t)$ and the objective function with H denoting the weight. The Lyapunov difference between consecutive time slots is given by

$$L(\Xi_{t+1}) - L(\Xi_t) = \frac{1}{2}[J_{t+1}^2 - J_t^2] + \frac{1}{2}[\sum_{i=1}^{N}(D(ES_{i,t}, ES_{i,t+1}, \beta_i)], \tag{33}$$

where

$$D(ES_{i,t}, ES_{i,t+1}, \beta_i) = (ES_{i,t} - \beta_i)^2 - (ES_{i,t} - \beta_i)^2.$$

Given

$$J_t = \max\{J_{t-1} - \alpha, 0\} + \frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}},$$

we have

$$J_{t+1}^2 - J_t^2 \leq 2J_t\left(\frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}} - \alpha\right) + 1 + \alpha^2. \tag{34}$$

We also have

$$D(ES_{i,t}, ES_{i,t+1}, \beta_i) \leq 2X_{i,t}(ES_{i,t} - \beta_i) + \max\{x_{i,\min}^2, x_{i,\max}^2\}. \tag{35}$$

Using Equations (37) - (39), we have

$$\begin{aligned}\Delta(\Xi_t) + H\mathrm{E}[\mathrm{CM_t}|\Xi_t] &= \frac{1}{2}(1+\alpha^2) + \frac{1}{2}\sum_{i=1}^{N}\max\{x_{i,\min}^2, x_{i,\max}^2\}\\ &+ J_t\left[\frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}} - \alpha|\Xi_t\right]\\ &+ \sum_{i=1}^{N}(ES_{i,t} - \beta_i)\mathrm{E}[\mathrm{X_{i,t}}|\Xi_t]\\ &+ H\mathrm{E}[\mathrm{CM_t}|\Xi_t],\end{aligned}$$

which upper bounds the drift-plus-cost or the objective function in M-(ST-OPT).

Proof of Theorem 1.1 This theorem proof deals with the ability to obtain system feasibility with MATCH. This involves showing the satisfaction of (i) constraint (3) and (ii) $ES_{i,t} \in [es_{i,\min}, es_{i,\max}]$. Under the Lyapunov optimization framework, it sufficient to show that the virtual queue J_t is mean rate stable, i.e., $\lim_{T\to\infty}\frac{\mathrm{E[J_i,T]}}{T} = 0$, for constraint (3) to be satisfied. This automatically follows from the result in Proposition 2.1, Section 4.4 from [29]. The structure of the proof to satisfy $ES_{i,t} \in [es_{i,\min}, es_{i,\max}]$ involves first proving a lemma followed by the use of mathematical induction.

Lemma 1. *If*

$$1.ES_{i,t} < -x_{i,\min}, X_{i,t}^{mst-opt} = \min\{\Phi_{i,t}^r RG_{i,t}, x_{i,\max}\}$$

and if

$$2.ES_{i,t} > \beta_i - H(p_{sell,\min} + DG'_{i,\min}), X_{i,t}^{mst-opt} = x_{i,\min}.$$

Proof. We transform M-(ST-OPT) as

$$\arg \min_{\{\overrightarrow{X_t}, L_{tot,t}, CEG_t, e_{sell,t}\}} OBJ_t,$$

where

$$\begin{aligned} OBJ_t &= \left[\sum_{i=1}^{N} H \cdot DG_i(X_{i,t}) + (ES_{i,t} - \beta_i)X_{i,t}\right] \\ &+ H \cdot CCG(CEG_t) \\ &+ H \cdot P_{buy,t} e_{buy,t}(e_{sell,t} + L_{tot,t} - CEG_t + \sum_{i=1}^{N} X_{i,t}) \\ &- H \cdot P_{sell,t} e_{sell,t} - \frac{J_t}{L_{e,t}} L_{tot,t}, \end{aligned}$$

subject to

$$L_{tot,t} \leq L_{c,t} + L_{e,t}, \forall t. \quad (36)$$

$$CG_{i,t} \in [0, cg_{\max}], \forall i, t. \quad (37)$$

$$CG_t = \Phi_t^c CEG_t, \forall t. \quad (38)$$

$$|CG_t - CG_{t-1}| \leq r \cdot cg_{\max}, \forall t. \quad (39)$$

$e_{sell,t} \geq 0$.

$$x_{i,\min} \leq X_{i,t} \leq \min\{\Phi_{i,t}^r RG_{i,t}, x_{i,\max}\}. \quad (40)$$

$$X_{i,t} \geq \sum_{i=1}^{N} \Phi_{i,t}^r RG_{i,t} - \sum_{j \neq i}^{N} X_{j,t} - L_{tot,t} + CEG_t - e_{sell,t}. \quad (41)$$

Since the objective function of the above optimization problem is separable over all variables, an optimal solution of $X_{i,t}$ can be derived through the following optimization problem.

$$\underset{X_{i,t}}{\operatorname{argmin}}\, H \cdot DG_i(X_{i,t}) + (ES_{i,t} - \beta_i)X_{i,t} + H \cdot P_{buy,t} X_{i,t},$$

subject to (44) and (45). Under the assumption that $ES_{i,t} < \beta i - H(p_{buy,\max} + DG'_{i,\max}) = -x_{i,\min}$, the objective function is strictly decreasing with respect to $X_{i,t}$. Thus, the optimal value of $X_{i,t}$ is $\min\{\Phi_{i,t}^r RG_{i,t}, x_{i,\max}\}$. In a similar fashion when $ES_{i,t} > \beta_i - H(p_{sell,\min} + DG'_{i,\min})$, the optimal solution to $X_{i,t}$ is achieved via the solution to the following optimization problem.

$$\underset{X_{i,t}}{\operatorname{argmin}}\, H \cdot DG_i(X_{i,t}) + (ES_{i,t} - \beta_i)X_{i,t} + H \cdot P_{sell,t} X_{i,t},$$

subject to (44) and

$$X_{i,t} \leq \sum_{i=1}^{N} \Phi_{i,t}^r RG_{i,t} - \sum_{j \neq i}^{N} X_{j,t} - L_{tot,t} + CEG_t + e_{buy,t}. \quad (42)$$

This objective function is strictly increasing with respect to $X_{i,t}$. Thus, the optimal solution to $X_{i,t}$ is $x_{i,\min}$. Thus, we have proved Lemma 1. ■
Using Lemma 1, we will not show using mathematical induction that the condition $ES_{i,t} \in [es_{i,\min}, es_{i,\max}]$ is satisfied. We have the following lemma in this regard.

Lemma 2. *The energy state for the i^th storage unit is bounded in the interval* $[0, es_{i,\max}]$.

Proof. For $t = 0$, we have $ES_{i,t} = 0$, which is bounded. In the inductive step we assume that $ES_{i,t} \in [0, es_{i,\max}]$, and need to show that $ES_{i,t+1} \in [0, es_{i,\max}]$. When $ES_{i,t} \in [0, -x_{i,\min}]$, using Lemma 1 and Equation (5), we have $ES_{i,t+1} = ES_{i,t} + \min\{\Phi_{i,t}^r RG_{i,t}, x_{i,\max} \geq ES_{i,t} \geq 0$. We also have $ES_{i,t+1} \leq ES_{i,t} + x_{i,\max} < es_{i,\max}$. When $ES_{i,t} \in [-x_{i,\min}, \beta_i - H(p_{sell,\min} + DG'_{i,\min}]$, using (5), we have $ES_{i,t+1} \in [ES_{i,t} + x_{i,\min}, ES_{i,t} + x_{i,\max}]$. From the expressions for β_i and H_{max}, it follows that $ES_{i,t+1} \in [0, es_{i,\max}]$. When $ES_{i,t} \in [\beta_i - H(p_{sell,\min} + DG'_{i,\min}, -x_{i,\max}]$, using Lemma 1 and Equation (5), we have $ES_{i,t+1} = ES_{i,t} + x_{i,\min} < ES_{i,t} \leq es_{i,max}$. Thus, we have proved Lemma 2. ■.
Using Lemmas 1 and 2, the result of Theorem 1.1 easily follows. ■.

Proof of Theorem 1.2 The theorem is proved via two lemmas.

Lemma 3. *Consider the following optimization problem.*

$$\textbf{\textit{OPT:}}\ \arg \min_{\{\overrightarrow{R_t}, \overrightarrow{X_t}, L_{tot,t}, CEG_t, e_{buy,t}, e_{sell,t}\}} \frac{1}{T} \sum_{t=0}^{T-1} \mathrm{E}[CM_t],$$

where

$$\overrightarrow{R_t} = \{R_{1,t}, R_{2,t}, ..., R_{N,t}\};\ \overrightarrow{X_t} = \{X_{1,t}, X_{2,t}, ..., X_{N,t}\}$$

subject to

$$L_{tot,t} \leq L_{c,t} + L_{e,t}, \forall t. \quad (43)$$

$$L_{c,t} \in [l_{c,\min}, l_{c,\max}];\ L_{e,t} \in [l_{e,\min}, l_{e,\max}], \forall t. \quad (44)$$

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathrm{E}\left[\frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}}\right] \leq \alpha. \quad (45)$$

$$X_{i,t} \in [x_{i,\min}, x_{i,\max}], \forall i, t. \quad (46)$$

$$R_{i,t} = \Phi_{i,t}^r (RG_{i,t} - \eta X_{i,t}),\ R_{i,t} \geq 0, \eta \in [0,1], \forall i, t, \quad (47)$$

where

$$\Phi_{i,t}^r \in [0,1], RG_{i,t} \in [0, rg_{i,\max}]$$

.

$$R_{i,t} \in [0, r_{i,\max}], \forall i, t. \quad (48)$$

$$CG_{i,t} \in [0, cg_{\max}], \forall i, t. \quad (49)$$

$$CG_t = \Phi_t^c CEG_t, \forall t, \quad (50)$$

where $\Phi_t^c \in [0,1]$; $CEG_t \in [0, \frac{cg_{max}}{\Phi_t^c}], \forall t$.

$$e_{buy,t} \geq 0, \forall t. \quad (51)$$

$$e_{sell,t} \geq 0, \forall t. \quad (52)$$

$$CG_t + e_{buy,t} + \sum_{i=1}^{N} R_{i,t} = e_{sell,t} + L_{tot,t}, \forall t. \quad (53)$$

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathrm{E}[X_{i,t}] = 0, \forall i. \quad (54)$$

There exists a stationary randomized solution to OPT that satisfies the following conditions.

$$\mathrm{E}[CM_t^s] \leq \tilde{cm}, \forall t, \quad (55)$$

$$\mathrm{E}[X_{i,t}^s] = 0, \forall i, t, \quad (56)$$

and

$$\frac{1}{T} \sum_{t=0}^{T-1} \mathrm{E}\left[\frac{L_{c,t} + L_{e,t} - L_{tot,t}^s}{L_{e,t}}\right] \leq \alpha, \forall t. \quad (57)$$

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Here, $\tilde{cm}$ is optimal system cost for OPT, and all the expectations are taken over the randomness of the system state and the possible randomness of the decisions.

Proof. The lemma can be easily derived using Theorem 4.5 in [29], which provides sufficient conditions for the existence of a stationary and randomized solution to OPT, and the fact that these conditions are met by the problem. ■

Minimizing the drift-plus-cost function $\Delta(\Xi_t) + H\mathrm{E}[\mathrm{CM_t}|\Xi_t]$, the real-time subproblem for OPT at time slot t is given by

M1-(ST-OPT): $\arg \min_{\{\overrightarrow{R_t}, \overrightarrow{X_t}, L_{tot,t}, CEG_t, e_{buy,t}, e_{sell,t}\}} OBJ_t,$

where

$$\begin{aligned} OBJ_t &= \left[\sum_{i=1}^{N} H \cdot DG_i(X_{i,t}) + (ES_{i,t} - \beta_i)X_{i,t}\right] \\ &+ H \cdot CCG(CEG_t) + H \cdot P_{buy,t}e_{buy,t} \\ &- H \cdot P_{sell,t}e_{sell,t} - \frac{J_t}{L_{e,t}}L_{tot,t}, \end{aligned}$$

where

$$H \in [0, H_{\max}];$$

$$H_{\max} = \min_{\forall i}\left\{\frac{ES_{i,\max} + X_{i,\min} - X_{i,\max}}{P_{buy,\max} - P_{sell,\min} + DG'_{i,\max} - DG'_{i,\min}}\right\},$$

$$\beta_i = H(P_{buy,\max} + DG'_{i,\max}) - X_{i,\min},$$

and

$$J_t = \max\{J_{t-1} - \alpha, 0\} + \frac{L_{c,t} + L_{e,t} - L_{tot,t}}{L_{e,t}}.$$

subject to

$$L_{tot,t} \le L_{c,t} + L_{e,t},\ \forall t. \tag{58}$$

$$L_{c,t} \in [l_{c,\min}, l_{c,\max}];\ L_{e,t} \in [l_{e,\min}, l_{e,\max}],\ \forall t. \tag{59}$$

$$X_{i,t} \in [x_{i,\min}, x_{i,\max}],\ \forall i, t. \tag{60}$$

$$R_{i,t} = \Phi^r_{i,t}(RG_{i,t} - \eta X_{i,t}),\ R_{i,t} \ge 0, \eta \in [0,1],\ \forall i, t, \tag{61}$$

where $\Phi^r_{i,t} \in [0,1]$, and $RG_{i,t} \in [0, rg_{i,\max}]$.

$$R_{i,t} \in [0, r_{i,\max}],\ \forall i, t. \tag{62}$$

$$CG_{i,t} \in [0, cg_{\max}], \forall i, t. \tag{63}$$

$$CG_t = \Phi^c_t CEG_t, \forall t, \tag{64}$$

where $\Phi^c_t \in [0,1]$; $CEG_t \in [0, \frac{cg_{max}}{\Phi^c_t}], \forall t$.

$$e_{buy,t} \ge 0,\ \forall t. \tag{65}$$

$$e_{sell,t} \ge 0,\ \forall t. \tag{66}$$

$$CG_t + e_{buy,t} + \sum_{i=1}^{N} R_{i,t} = e_{sell,t} + L_{tot,t},\ \forall t. \tag{67}$$

This problem is the same as M-(ST-OPT) except the inclusion of the ramping constraint. Denote the optimal objective values at time slot t of M-(ST-OPT) and M1-(ST-OPT) by $\tilde{opt}_t$ and opt^*_t respectively. Denote the corresponsing argument variables by $\overrightarrow{\tilde{arg}_t}$ and $\overrightarrow{arg^*_t}$ respectively. We then have the following lemma.

Lemma 4. *At each time slot t, opt^*_t is bounded as*

$$\tilde{opt}_t \le opt^*_t \le \tilde{opt}_t + \epsilon,$$

where

$$\epsilon = H(1-r)cg_{\max}\max\{p_{buy,\max}, CCG'_{max}\}.$$

Proof. Since M-(ST-OPT) is more restricted than M1-(ST-OPT), we have $\tilde{opt}_t \le opt^*_t$. Now we proceed to upper bound $opt^*_t - \tilde{opt}_t$. The proof structure consists of bounding $opt^*_t - \tilde{opt}_t$ when (i) $cg^*_t = \tilde{cg}_t$, (ii) $cg^*_t < \tilde{cg}_t$, and (iii) $cg^*_t > \tilde{cg}_t$. For (i) it is trivial to prove that $opt^*_t = \tilde{opt}_t$. In the case of $cg^*_t < \tilde{cg}_t$, the following holds.

$$\max\{CG_{t-1} - r \cdot cg_{\max}, 0\} \le CG_t \le CG_{t-1} + r \cdot cg_{\max}.$$

Set $\hat{opt}_t = \{\overrightarrow{R_t}, \overrightarrow{X_t}, L_{tot,t}, CG_{t-1} + r \cdot cg_{max}, e_{\tilde{buy},t} + \tilde{CG}_t - CG_{t-1} - r \cdot cg_{\max}, e_{\tilde{sell},t}\}$ as a feasible solution to M-(ST-OPT). Thus $\hat{opt}_t$ is the same as $\tilde{opt}_t$ except the solutions of CG_t and $e_{buy,t}$. Intuitively, $\hat{opt}_t$ results from the fact that due to the ramping constraint, the conventional energy source is forced to generate less energy, and the aggregator chooses to buy more energy from the external energy markets to balance power. The upper bound of the gap $opt^*_t - \tilde{opt}_t$ for case (ii) is given by

$$\begin{aligned} opt^*_t - \hat{opt}_t &\le H \cdot p_{buy,t}(\tilde{CG}_t - CG_{t-1} - r \cdot cg_{max}) && (68)\\ &\le H(1-r)cg_{max}p_{buy,\max}, && (69) \end{aligned}$$

where the inequality in (76) holds since $\tilde{CG}_t > CG_{t-1} + r \cdot cg_{max}$, and the function $CCG(\cdot)$ is non-decreasing. Thus, from (77) we have

$$opt^*_t - \tilde{opt}_t \le \hat{opt}_t - \tilde{opt}_t \le H(1-r)cg_{max}p_{buy,\max} \tag{70}$$

In the case of (iii) when $cg^*_t > \tilde{cg}_t$, we have

$$CG_{t-1} - r \cdot cg_{\max} \le CG_t \le CG_t \le \min\{cg_{max}, CG_{t-1} + r \cdot cg_{\max}\}.$$

Set $\hat{opt}_t = \{\overrightarrow{R_t}, \overrightarrow{X_t}, L_{tot,t}, CG_{t-1} - r \cdot cg_{max}, e_{\tilde{buy},t}, e_{\tilde{sell},t} - \tilde{CG}_t + CG_{t-1} - r \cdot cg_{\max}, \}$ as a feasible solution to M-(ST-OPT). Thus $\hat{opt}_t$ is the same as $\tilde{opt}_t$ except the solutions of CG_t and $e_{buy,t}$. Intuitively, $\hat{opt}_t$ results from the fact that due to the ramping constraint, the conventional energy source is forced to generate more energy, and the aggregator chooses to sell more energy to the external energy markets to balance power. The upper bound of the gap $opt^*_t - \tilde{opt}_t$ for case (ii) is given by

$$\begin{aligned} opt^*_t - \hat{opt}_t &\le H[CCG(CG_{t-1} - rcg_{max}) - CCG(\tilde{CG}_t)] && (71)\\ &\le H(CG_{t-1} - r \cdot cg_{max} - \tilde{CG}_t)CCG'_{\max} && (72)\\ &\le H(1-r)cg_{max}CCG'_{\max}, && (73) \end{aligned}$$

where the inequality in (79) holds since $\tilde{CG}_t < CG_{t-1}r \cdot cg_{max}$, and (80) is derived through the mean value theorem. Thus, from (81) we have

$$opt^*_t - \tilde{opt}_t \le \hat{opt}_t - \tilde{opt}_t \le H(1-r)cg_{max}CCG'_{\max} \tag{74}$$

Combining (78) and (82), we get

$$opt^*_t - \tilde{opt}_t \le H(1-r)cg_{\max}\max\{p_{buy,\max}, CCG'_{\max}\},$$

which completes the proof of Lemma 4. ■

Using the result for the upper bound of the cost-plus-drift function, and Lemmas 3 and 4, we can upper bound the cost-plus-drift function using the MATCH algorithm as follows.

$$\begin{aligned} \Delta(\Xi_t) + H\mathrm{E}[\mathrm{CM}^*_t|\Xi_t] \le B + \epsilon \\ + J_t\left[\frac{L_{c,t} + L_{e,t} - \tilde{L_{tot,t}}}{L_{e,t}} - \alpha|\Xi_t\right] \\ + \sum_{i=1}^{N}(ES_{i,t} - \beta_i)\mathrm{E}[\tilde{X_{i,t}}|\Xi_t] \\ + H\mathrm{E}[\tilde{\mathrm{CM}}_t|\Xi_t], \end{aligned} \tag{75}$$

or

$$\begin{aligned}\Delta(\Xi_t) + H\mathrm{E}[\mathrm{CM}_t^*|\Xi_t] \le B + \epsilon \\ + J_t\left[\frac{L_{c,t}+L_{e,t}-L_{tot,t}^{\tilde{s}}}{L_{e,t}} - \alpha|\Xi_t\right] \\ + \sum_{i=1}^{N}(ES_{i,t}-\beta_i)\mathrm{E}[\mathrm{X}_{i,t}^{\tilde{s}}|\Xi_t] \\ + H\mathrm{E}[\mathrm{C\tilde{M}}_t^s|\Xi_t],\end{aligned} \tag{76}$$

or

$$\Delta(\Xi_t) + H\mathrm{E}[\mathrm{CM}_t^*|\Xi_t] \le B + \epsilon + H\cdot CM^{st-opt}. \tag{77}$$

Here

$$B = \frac{1}{2}(1+\alpha^2) + \frac{1}{2}\sum_{i=1}^{N}\max\{x_{i,\min}^2, x_{i,\max}^2\}. \tag{78}$$

Here (83) is derived via Lemmas 1 to 4, (84) holds since M1-(ST-OPT) minimizes the righthand side of (83), (85) is based on Equations (61) - (63) in Lemma 3, and the fact that the stationary solution to M-(ST-OPT) is independent of Ξ_t, and (88) holds since OPT is a relaxed version of ST-OPT. Taking expectations over Ξ_t on both sides of (85), and summing over all time slots, we get

$$\mathrm{E}[L(\Xi_t)] - L(\Xi_t) + H\sum_{t=0}^{T-1}\mathrm{E}[CM_t^*] \le (B+\epsilon+H\cdot CM^{st-opt}. \tag{79}$$

Since $L(\Xi_t)$ is non-negative, we can re-write (87) as

$$\frac{1}{T}\sum_{t=0}^{T-1}\mathrm{E}[CM_t^*] \le \frac{B+\epsilon+H\cdot CM^{st-opt}}{H} + \frac{L(\Xi_t)}{TH}. \tag{80}$$

Taking the limit superior on both sides of the equation gives us the relation

$$CM^* - CM^{st-opt} \le \frac{B}{H} + (1-r)cg_{\max}\max\{p_{buy,\max}, CCG'_{max}\}$$

Thus, we have proved Theorem 1.2. ■

Proof of Theorem 1.3 The lower bound on $CM^{st-opt}(r)$ is derived by setting $r=1$ in Theorem 1.2, and using the fact that $CM^{st-opt}(1) \le CM^{st-opt}(r)$. Hence we have proved Theorem 1.1. ■

Proof of Theorem 1.4 The proof follows directly from Lemmas 1 and 2 with the application of mathematical induction. ■

Proof of Theorem 1.5 We use the principle of mathematical induction for our proof. For $t=0$, we have $J_t = 0$, which is upper bounded. In the inductive step, assuming $J_t \le Hp_{buy,\max}l_{e,\max}+1$, we need to show $J_{t+1} \le Hp_{buy,\max}l_{e,\max}+1$. When $J_t \le Hp_{buy,\max}l_{e,\max}$, we have

$$J_{t+1}+1 \le \max\{J_t-\alpha, 0\}+1 \le J_t+1 \le Hp_{buy,\max}l_{e,\max}+1.$$

When $J_t \in (Hp_{buy,\max}l_{e,\max}, Hp_{buy,\max}l_{e,\max}+1)$, we need to show that the unique solution to M-(ST-OPT) is $L_{c,t}+L_{e,t}$. In that case we would have

$$J_{t+1} = \max\{J_t-\alpha, 0\} \le J_t \le Hp_{buy,\max}l_{e,\max}+1.$$

Consider the optimization problem M′-(ST-OPT). We fix the variables $(\overrightarrow{X_t}, CEG_t, e_{sell,t})$, and minimize M′-(ST-OPT) over $L_{tot,t}$. The optimal solution $L_{tot,t}$ can be arrived at through the following problem.

$$\arg\min_{L_{tot,t}}\left(Hp_{buy,t} - \frac{J_t}{L_{e,t}}\right)L_{tot,t}$$

subject to

$$L_{c,t} \le L_{tot,t} \le L_{c,t} + L_{e,t},$$

$$L_{tot,t} \ge \sum_{i=1}^{N}(R_{i,t}) + CEG_t - e_{sell,t}.$$

When $J_t > Hp_{buy,\max}l_{e,\max}$, the objective function is strictly decreasing. Thus, the optimal solution of $L_{tot,t}$ is $L_{c,t}+L_{e,t}$. Suppose that the optimal solutions of $e_{buy,t}$ and $e_{sell,t}$ satisfy $e_{buy,t}^{mst-opt} \ge e_{sell,t}^{mst-opt} > 0$. We can then show that there is another feasible solution $\{\overrightarrow{R_t^{mst-opt}}, \overrightarrow{X_t^{mst-opt}}, L_{tot,t}^{mst-opt}, CEG_t^{mst-opt}, e_{buy,t}^{mst-opt} - e_{sell,t}^{mst-opt}, 0\}$, achieving a strictly smaller objective value, hence contradicting the fact that $\{\overrightarrow{R_t^{mst-opt}}, \overrightarrow{X_t^{mst-opt}}, L_{tot,t}^{mst-opt}, CEG_t^{mst-opt}, e_{buy,t}^{mst-opt}, e_{sell,t}^{mst-opt}\}$ is optimal. The proof for the case when $e_{sell,t}^{mst-opt} \ge e_{buy,t}^{mst-opt} > 0$ is similar and omitted here. Hence we have proved Theorem 1.5. ■

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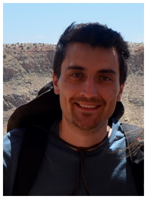

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