Efficient Charging of Access Limited Wireless Underground Sensor Networks

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Abstract—Wireless underground sensor networks (WUSNs) present a variety of new research challenges. Magnetic induction (MI) based transmission has been proposed to overcome the very harsh propagation conditions in underground communications in recent years. In this approach, induction coils are utilized as antennas in the sensor nodes. This solution achieves larger transmission ranges compared to the traditional electromagnetic (EM) waves based approach. In the past, some efforts have been made to characterize the signal transmission in MI-WUSNs. Those investigations, however, refer mostly to the information transmission. One of the open issues, that may constrain the system design in some of the applications, is the powering of the individual sensor nodes. Due to the low accessibility of the nodes, a new method of wireless power transfer (WPT) for MI-WUSNs is proposed in this work. This method is mainly based on simultaneous signal transmissions from multiple sensor nodes with optimized signal constellations. Furthermore, the optimal scheduling for power transmission and reception is provided, which maximizes the energy efficiency of the network charging procedure. The proposed method is compared with the naive approaches and shows a significant improvement of the system performance in terms of energy efficiency.

Index Terms—Underground communications, wireless sensor networks, wireless charging, inductive power transmission.

I. INTRODUCTION

The objective of Wireless Underground Sensor Networks (WUSNs) is to establish an efficient wireless communication in the challenging underground medium. Typical applications for such networks include soil condition monitoring, earthquake prediction, communication in mines/tunnels, etc. [2], [3]. Due to the harsh propagation conditions in the soil medium (including rock, sand, and water sheds), traditional wireless signal propagation techniques using electromagnetic (EM) waves can only be applied for very short transmission ranges due to a high path loss and vulnerability to changes of soil properties, such as moisture [4], [5]. Magnetic induction (MI) based WUSNs were first introduced in [3], [6] and make use of magnetic antennas implemented as coils. This technique has been shown to be less vulnerable to the losses in conductive medium, such that the transmission range and coverage of the sensor network can be significantly improved by using MI based transceivers. So far, most of the previous works aimed at the investigation of the potential and problems of MI-WUSNs from the perspective of the information transmission. For example, some efforts have been made to characterize the channel capacity of a point-to-point signal transmission [6], [7], and the network throughput of tree-based MI-WUSNs [8], and to design the digital signal processing in MI-WUSNs [9]. Rescue and disaster aware MI-WUSNs have been proposed in [10], [11]. In parallel to the theoretical investigations, some research groups have conducted experiments in order to verify the correctness of the typical assumptions in the most common system models, e.g. [10], [12], [13], [14].

An important issue in sensor networks is the battery lifetime [2]. In many applications of WUSNs, the nodes can be charged wirelessly [15], especially if the charging device can be moved closely enough to the sensor node. In such cases, the wireless
power transfer (WPT) for traditional WSNs can be directly applied to the WUSNs with a slight change due to the medium characteristics and transceiver design (coils instead of RF antenna). For this, the recently proposed concepts of multiple-input multiple-output (MIMO) based WPT might be useful [12], [16], [17]. The mentioned works assume a single transmitter equipped with several resonant coils. Typically, the optimization of the coil configuration and system parameters refers to impedance matching and search for the optimal carrier frequency, for which the WPT efficiency is maximized [12], [16]. In [17], a so-called 3D-coil based transmitter is used in order to maximize the WPT efficiency in presence of multiple receivers. A novel beamforming solution is obtained, which is especially power efficient for scenarios with strong couplings between all transceivers and random orientation of the receiver coils. The major benefit of this approach is the reduced system complexity, since no hardware (e.g. impedance) adjustments need to be performed. On the contrary, the actual beamforming is achieved via optimal signal processing of the transmit signals, similarly to the traditional RF beamforming. Unfortunately, this beamforming solution is not applicable for MI-WUSNs due to very weak couplings between sensor nodes.

In modern applications of WSNs, in order to charge all sensor nodes, a mobile charging vehicle with optimized charging parameters is utilized [18], [19], as already mentioned before. However, several applications of WUSNs require the deployment of sensor nodes in hardly accessible (extreme) environments, where a mobile vehicle may not be able to get close enough to each node for charging. This situation occurs e.g. in mines, where the tunnels may not always be suitable for the mobile charger to freely move along, in oil reservoirs [20], and in structural health monitoring [21]. Even if some of the nodes can be accessed by the charger, the energy/fuel consumption by the mobile vehicle may become one of the crucial parameters for the overall exploitation costs. Alternatively, a stationary power source is utilized, which can be deployed above-ground and have a wireless or wired connection to one of the sensor nodes. This node is then used to wirelessly charge the whole network. Therefore, one of the sensor nodes (close to the edge of the deployment field) is selected as master node with sufficient power supply. Hence, the focus of this work is on the optimization of the corresponding charging procedure. One possible solution (naive approach) is a transmission solely from the master node [22], until all batteries are charged. However, more advanced techniques can heavily increase the energy efficiency. In particular, the concept of relayed energy transfer in sensor networks seems promising, especially the recently proposed multi-hop energy transfer [23], [24]. Here, the energy is guided via intermediate nodes, which are placed between the energy source and the target node. Since all sensor nodes are typically assumed to be equipped with only one antenna, omnidirectional signal propagation results, such that heavy energy losses are inevitable. On the other hand, the problem of optimal charging can be viewed as a beamforming problem for a distributed MIMO system, where each node represents a part of a large scale antenna array. Hence, a beamforming gain can be expected, if multiple nodes transmit simultaneously and their complex amplitudes are optimized to overlap constructively, especially in case of multiple receivers [17]. Then, all other sensor nodes may contribute to the efficient charging of each particular sensor node. Of course, one of the necessary conditions for establishing such a distributed MIMO system is the low mobility of sensor nodes, which is valid for WUSNs due to the stationary deployment. For a fair comparison between the different charging procedures, the definition of energy efficiency known from the literature (e.g. [25]) cannot be utilized, since different sensor nodes may require different amounts of energy. Hence, we define the energy efficiency as the ratio of the total amount of required energy for all sensor nodes in the network and the total consumed energy in the master node, which is needed in order to charge all sensor nodes. With this definition, the most energy efficient scheduling of signal transmissions is established in this work. Our main contribution is a novel power transmission policy, which aims at minimizing the overall energy losses of the master node. In particular, a non-linear non-convex optimization problem is formulated, which is split up in two subproblems by relaxing some of the constraints. Both problems are solved via iterative algorithms. The resulting
solution provides a close-to-optimum performance. A significant enhancement of the energy efficiency of the charging procedure is observed compared to the naive approach. This makes the proposed scheme very promising for the considered scenarios. Furthermore, with increasing numbers of simultaneous transmissions per time slot, the resulting energy efficiency converges very fast. Hence, a trade-off between system complexity and energy efficiency is discussed as well.

This paper is organized as follows. Section II describes the system model, which includes the effects resulting from deployment of the sensor nodes close to the ground surface onto the signal transmission. The novel technique of distributed MIMO based multihop energy transfer for MI-WUSNs is presented in Section III. Section IV provides numerical results and Section V concludes the paper.

II. SYSTEM MODEL

As mentioned earlier, we assume that one of the sensor nodes (master node) is connected to the power source in order to provide enough power supply for the other nodes of the network, see Fig. 1. Obviously, there are different ways of how the power signals from the master node can reach the target nodes. Hence, a relayed power transfer with possible retransmissions seems promising.

Each circuit includes a magnetic antenna (an air core coil) with inductivity $L$, a capacitor with capacitance $C$, a resistor $R$ (which models the copper resistance of the coil), and a load resistor $R_L$, see Fig. 2. The capacitor is designed to make the circuits resonant at the carrier frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$. The load resistor $R_L$ is chosen to minimize the power reflection at the receiver. The optimal value for the load resistor in MI-WUSNs $R_L = R$ is selected according to [8]. As argued in the previous works (e.g. [8]), the passive circuit elements need to be identical for all sensor nodes, in order to reduce the manufacturing costs and simplify the system design. As it has been mentioned in [11], the problem of frequency splitting does not arise in MI-WUSNs due to relatively weak couplings between the coils. Hence, we can assume that the optimal power efficiency is reached at the resonance frequency $f_0$. Then, the complex impedance of the capacitor compensates the impedance of the coil completely, such that the inner impedance of each circuit is given by $Z_{in} = R + R_L$. The induced voltage is related to the coupling between the coils, which is determined by the mutual inductance, denoted as $M_{k,l}$ for coils $k$ and $l$ [8],

$$M_{k,l} = \mu \pi N_w^2 \frac{\alpha^4}{4 \sqrt{r_{k,l}}} J \cdot G_{k,l},$$

where $r_{k,l}$ denotes the distance between the coils, $\alpha$ stands for the coil radius, $N_w$ is the number of windings, and $\mu$ denotes the permeability of the medium. For the polarization factor $J$, due to the assumed vertical axes deployment, $J = 1$ holds [11], and $G_{k,l}$ stands for the additional signal attenuation in the conductive medium. As discussed in previous works (e.g. [14], [26]), signal transmission within the underground medium suffers from a high path loss due to the conductive property of the soil. In order to simplify the investigations of the MI-WUSNs, all sensor nodes are typically assumed to be deployed deeply in the soil [8]. Then, a large part of the magnetic field propagates directly through the medium and suffers from the effect of eddy currents. However, if the nodes are deployed close to the ground surface, the magnetic field of the coil may penetrate the surface and be less vulnerable to the eddy currents. Obviously the burial depth can be

\[2\] For the WPT, only a single frequency is utilized [25].
omitted from the theoretical analysis, if it is much less than the distance between adjacent coils. This is a valid assumption as also recognized by [27]. Hence, we consider a deployment exactly at the ground surface. In order to incorporate this effect into the system model, we utilize the results from [27], [28]. Then, \( G_{k,l} \) is modeled by

\[
G_{k,l} = \frac{2}{\gamma_{k,l}^2} \left( 9 - (9 + 9\gamma_{k,l} + 4\gamma_{k,l}^2 + \gamma_{k,l}^3) e^{-\gamma_{k,l}} \right) \tag{2}
\]

instead of the well-known expression [9]

\[
G_{k,l} = e^{-\gamma_{k,l}}, \tag{3}
\]

which is valid for a transmission through the soil in case of a very large burial depth. Here, \( \gamma_{k,l} = r_{k,l}/\sqrt{\pi f_0 \sigma} \), where \( \sigma \) stands for the conductivity of the soil. It can be shown that the mutual inductance in [28] based on (2) corresponds to (1) with \( G_{k,l} \) from (3) for an identical conductivity in soil and air. This indicates the correct use of the system model from [28] in this work.

For the following, we define \( Z_{k,l} = j^2\pi f_0 M_{k,l}, \forall k \neq l \). Due to the symmetry of the magnetic coupling, \( Z_{k,l} = Z_{l,k}, \forall k, l \) holds. We consider the complex-valued amplitudes \( U_k \) and \( I_k \) of the voltages \( u_k(t) = U_k \cdot e^{2\pi f_0 t} \) and currents \( i_k(t) = I_k \cdot e^{2\pi f_0 t}, \forall k \), respectively. For each coil \( k \), the current amplitude \( I_k \) in the resonant circuit depends on the current amplitudes \( I_l, \forall l \neq k \) in all surrounding circuits via the voltage equation

\[
I_k \cdot Z_{in} + \sum_{l \neq k} (I_l \cdot Z_{k,l}) = U_k. \tag{4}
\]

In the following, the superscripts \((\cdot)^T\) and \((\cdot)^H\) denote transpose and Hermitian transpose, respectively. Furthermore, \( tr(\cdot) \) and \( \text{rank}(\cdot) \) represent the trace and the rank operator, respectively. In order to calculate the currents in all circuits of the coupled network, a set of voltage equations \( \mathbf{Z} \cdot \mathbf{I}_c = \mathbf{U} \) needs to be solved with respect to the current vector \( \mathbf{I}_c \).

Here, \( \mathbf{U} \) is the complex-valued input voltage vector. Both vectors, \( \mathbf{I}_c \) and \( \mathbf{U} \) are of length \( N_{\text{nodes}} \), which corresponds to the number of sensor nodes in the network. The impedance matrix \( \mathbf{Z} \) is defined as

\[
\mathbf{Z} = \begin{bmatrix}
Z_{in} & Z_{1,2} & \cdots \\
Z_{2,1} & Z_{in} & \cdots \\
\vdots & \vdots & \ddots
\end{bmatrix}. \tag{5}
\]

The solution for this set of equations is given by \( \mathbf{I}_c = \mathbf{Z}^{-1}\mathbf{U} \). The received power at the load resistor of node \( l \) is equal to

\[
P_{r,l} = |I_l|^2 R_L = |e_l^T \mathbf{Z}^{-1}\mathbf{U}|^2 R_L = \mathbf{U}^H (\mathbf{Z}^{-1})^H e_l R_L e_l^T \mathbf{Z}^{-1} \mathbf{U} = \mathbf{U}^H \mathbf{K}_l \mathbf{U}, \tag{6}
\]

where \( e_l = [0, \ldots, 0, 1, 0, \ldots, 0]^T \) with '1' at the \( l \)th position, and with implicit definition of matrix \( \mathbf{K}_l \). As shown in [17], the transmit power for each magnetic device \( k \) in case of weak couplings between coils can be approximated by

\[
P_{t,k} \approx |U_l|^2 \cdot Z_{in}^{-1} = \mathbf{U}^H e_l Z_{in}^{-1} e_l^T \mathbf{U} = \mathbf{U}^H \mathbf{G}_l \mathbf{U}, \tag{7}
\]

with implicit definition of matrix \( \mathbf{G}_l \). This approximation is valid for MI-WUSNs [8].

In this work, we assume a stationary deployment of the WUSN which is a valid assumption due to the low mobility of the sensor nodes deployed in underground medium. Hence, a nearly perfect synchronization\(^3\) of the nodes can be achieved using the commonly known methods of WSN synchronization [29], [30]. Furthermore, we assume that the channel state information (CSI) is available to the system designer, such that an optimization of the system parameters, the scheduling, and the beamforming solution can be conducted offline. The consumed energy due to the optimization itself is a constant value and negligible in case of multiple cycles of network charging.

### III. WPT FOR ACCESS LIMITED MI-WUSNS

In general, WPT for access limited WUSNs can be done via relaying of the power signals from node to node, until the target node is charged. In this case, only one node is scheduled for transmission per time slot. Interestingly, if multiple nodes are scheduled for transmission in the same time slot, this relayed powering can be viewed as beamforming for a distributed MIMO system with multiple time slots, where each time slot has its own beamforming pattern\(^4\). For this, we assume that all sensor nodes are perfectly synchronized. Hence, the beamforming patterns can be jointly optimized for all time slots.

\(^3\)The precision of synchronization is limited by the Cramer-Rao-Bound, as known from the theory of receiver synchronization. However, due to the stationary deployment, the channel state remains unchanged over a long period of time, such that very low estimation errors for the signal parameters like frequency, phase, etc. result and do not affect the beamforming algorithms noticeably.

\(^4\)We follow the convention of the literature on WPT using electromagnetic waves [25] and adopt the term beamforming for the optimization of the transmit signal vector in spatial domain.
in order to minimize the overall energy losses and to increase the energy efficiency of the charging. Keeping this in mind, we formulate the optimization problem in Section III-A and present a possible solution for this problem in Sections III-B-III-D.

A. Problem formulation

We assign index \( l = 1 \) to the master node. Hence, the optimization problem can be formulated as follows:

\[
\begin{align*}
\arg \min_{U(n), \forall 1 \leq n \leq N_{ts}} & \quad \sum_{n=1}^{N_{ts}} U(n)^H G_l U(n), \\
\text{s.t.:} & \quad 1) \sum_{n=1}^{N_{ts}} U(n)^H K_l U(n) \\
& \quad - \sum_{n=1}^{N_{ts}} U(n)^H G_l U(n) \geq b_l, \quad 1 < l \leq N_{\text{nodes}}, \\
& \quad 2) U(m)^H G_l U(m) \leq \sum_{n=1}^{m-1} U(n)^H K_l U(n) \\
& \quad - \sum_{n=1}^{m-1} U(n)^H G_l U(n), \quad 1 \leq m \leq N_{ts}, 1 < l \leq N_{\text{nodes}}, \\
& \quad 3) U(m)^H K_l U(m) \leq b_l - \sum_{n=1}^{m-1} U(n)^H K_l U(n) \\
& \quad + \sum_{n=1}^{m-1} U(n)^H G_l U(n), \quad 1 \leq m \leq N_{ts}, 1 < l \leq N_{\text{nodes}}, \\
& \quad 4) \text{nnz}(U(m)) \leq N_{\text{simultan}}, \quad 1 < m \leq N_{ts},
\end{align*}
\]

where \( U(n) \) represents the transmission vector in time slot \( n \), \( N_{ts} \) stands for the total number of time slots, and \( b_l \) is the required battery charge for the node \( l \). The first constraint 1) indicates that all nodes are supposed to be fully charged within \( N_{ts} \) time slots\(^5\). In this work, \( N_{ts} \) is not restricted, since a stationary deployment for the WUSNs can be assumed [11]. The second constraint 2) reflects the need for a sufficient battery level of all nodes before starting the transmission in time slot \( m \). Similarly, the third constraint 3) indicates that the received energy in time slot \( m \) cannot exceed the battery capacity. In particular, the right hand side of the inequality represents the remaining charge, which can be received before the battery reaches its maximum capacity. Note, that the nodes are not able to charge their batteries during their own transmissions, i.e., only if their transmit voltage is zero, a reception is possible. This is due to the difference between the receiving and the transmitting circuits [25], which cannot be connected to the battery at the same time. Therefore, we introduce the conditions \( e_l^T U(n) = 0 \) and \( e_l^T U(n) \neq 0 \) in the sums in (8). In addition, we denote the number of non-zero elements of a vector by \( \text{nnz}(\cdot) \) and the maximum number of nodes that can be selected for transmission in the same time slot by \( N_{\text{simultan}} \leq N_{\text{nodes}} \). Using this notation, the fourth constraint 4) is introduced in order to restrict the number of simultaneous transmissions. This restriction is applied in order to reduce the complexity of the system. In this work, we first provide a solution for \( N_{\text{simultan}} = 2 \) in Sections III-B-III-D and then show, how the proposed solution can be extended for \( N_{\text{simultan}} > 2 \). Of course, with larger values of \( N_{\text{simultan}} \), a better system performance and power transfer efficiency can be expected. However, the complexity of the problem increases exponentially in \( N_{\text{simultan}} \), such that the accuracy of a suboptimum solution might reduce.

In general, problem (8) is a non-linear non-convex program. This can be deduced directly from the case of low couplings between coils (which is a usual assumption in underground communications), where the influence of matrix \( K_l \) can be neglected. Then, using the fact that matrix \( G_l \) is positive semidefinite, it is observed that the constraint 3) is non-convex, which makes the whole optimization problem non-convex. As mentioned earlier, one feasible approach (fulfilling all constraints) corresponds to the possibility to transmit solely from the master node (our baseline scheme), until all batteries are charged. In the following, more advanced techniques are proposed.

B. General remarks

For \( N_{\text{simultan}} = 2 \), at most two nodes can be selected for transmission in each time slot. If only one node is selected, the corresponding assignment is referred to as 'single node transmission' (SNT). Due to the non-convexity of problem (8), it is not possible to use the well-known tools of convex

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\(^5\)As a consequence of all constraints, the last transmission would fulfill the first constraint with equality.
optimization [31]. Hence, we provide a suboptimal solution by splitting the problem in two parts. The first subproblem is a non-convex quadratically constrained quadratic program (QCQP), as described in Section III-C, which is formulated by relaxing the second and the third constraints of (8) and approximately solved using an iterative algorithm. The solution is the optimal beamforming pattern for each pair of nodes and power allocation for the SNTs. The second subproblem (described in Section III-D) is related to the transmission policy using the results from the first subproblem. In particular, the pairs and SNTs are assigned to all time slots.

C. Beamforming

Due to the mentioned constraint relaxation in the first subproblem, the optimization problem becomes independent from the particular time slot assignments. Intuitively, the beamforming solution (orientation of the vector \( \mathbf{U}(n) \)) for a given pair remains unchanged throughout the charging procedure, such that only the transmit energy may vary from time slot to time slot whilst the beamforming orientation remains the same. This property of the optimal solution is described by Theorem 1.

**Theorem 1.** If \( \mathbf{U}(n_1) \) is the optimal beamforming solution of (8) for pair \( x \) in time slot \( n_1 \) and \( \mathbf{U}(n_2) \) is the optimal beamforming solution for pair \( x \) in time slot \( n_2 \), then \( \mathbf{U}(n_1) = c \cdot \mathbf{U}(n_2) \) holds with a real positive constant \( c \).

The proof for this theorem can be found in Appendix. In total, there are \( N_{\text{pairs}} = N_{\text{nodes}}(N_{\text{nodes}}-1)/2 \) pairs and \( N_{\text{nodes}} \) SNTs. We introduce the complex voltage \( U_{s,x} \) of node \( s \) transmitting in pair \( x \). For \( x = 0 \), \( U_{s,0} \) corresponds to the SNT of node \( s \). In addition, vector \( \mathbf{V} = [V_1^T, V_2^T]^T \) is defined, where vector \( V_1 = [U_{1,0}, U_{2,0}, \ldots, U_{N_{\text{nodes}},0}]^T \) contains the voltages for SNTs and \( V_2 = [[U_{1,1}, U_{2,1}], \ldots, [U_{N_{\text{nodes}}-1,N_{\text{pairs}},s}, U_{N_{\text{nodes}},N_{\text{pairs}}},s]]^T \) contains the beamforming vectors for the node pairs. Hence, the total number of complex variables containing the voltage amplitudes is equal to \( 2N_{\text{pairs}} + N_{\text{nodes}} = N_{\text{nodes}}^2 \). This number corresponds to the length of vector \( \mathbf{V} \).

Assuming that the node pair \( x \) with nodes \( \{s_1(x), s_2(x)\} \) starts transmitting in time slot \( q \), the total transmitted energy from node \( s_1(x) \) using only pair \( x \) is given by

\[
\sum_{n=1}^{N_{\text{nodes}}} U^H(n) G_{s_1(x)} U(n)
\]

where \( W_x = E_{s_1(x),N_{\text{nodes}}+2x-1} + E_{s_2(x),N_{\text{nodes}}+2x} \) and \( E_{g_1,g_2} \) stands for a matrix of size \( N_{\text{nodes}} \times N_{\text{nodes}} \) with all zero elements except for ‘1’ at the position \((g_1, g_2)\), and \( \alpha_x \) has been defined implicitly. Here, \( \mathbf{U}(q) / \alpha_x \) is included in vector \( \mathbf{V} \), such that \( \mathbf{W}_x \mathbf{V} = \mathbf{U}(q) / \alpha_x \). For single node transmissions from node \( s \), \( E_{s,s} \) is utilized instead of \( \mathbf{W}_s \). Hence, the variables can be exchanged \((\mathbf{U}(n) \rightarrow \mathbf{V})\) by summing up transmit and receive signals\(^7\) over all \( x \), such that (8) is reformulated:

\[
\arg\min_{\mathbf{V}} \mathbf{V}^H \mathbf{D} \mathbf{V}, \text{ s.t.: } \mathbf{V}^H \mathbf{F}_l \mathbf{V} \geq b_l, 1 \leq l \leq N_{\text{nodes}},
\]

where we define the matrices \( \mathbf{D} \) and \( \mathbf{F}_l \) as

\[
\mathbf{D} = \sum_{x=1}^{N_{\text{pairs}}} W_x^T G_x W_x + E_{1,1}^T G_1 E_{1,1},
\]

\[
\mathbf{F}_l = \sum_{x=1}^{N_{\text{pairs}}} W_x^T K_x W_x + \sum_{s=1, s \neq l}^{N_{\text{nodes}}} E_{s,s}^T K_s E_{s,s}
\]

\[
- \sum_{x=1}^{N_{\text{pairs}}} W_x^T G_x W_x - E_{l,1}^T G_1 E_{l,1}.
\]

As previously assumed, the nodes are not able to charge their batteries during own transmission. This has been taken into account in (12) by restricting

\(^6\)Although the transmission starts at time slot \( q \), not all subsequent time slots may be used for transmission by pair \( x \), which is reflected in the condition \( e_{s_1(x)}^2 U(n) \neq 0, e_{s_2(x)}^2 U(n) = 0 \) for the summation in (9). Also, we exploit, that the beamforming pattern remains unchanged and only the relative amplification given by \( e_{s_1(x)}^2 U(n) / e_{s_1(x)}^2 U(q) \) varies among all time slots \( n \).

\(^7\)This can be done, since pairs use disjoint time slots.
the indices of the sums with \( l \notin \{ s_1(x), s_2(x) \} \) and \( l \in \{ s_1(x), s_2(x) \} \) for the node pairs. For the SNTs, we use \( s = l \) and \( s = l^8 \).

Obviously, (10) is non-convex due to the non-convex constraints. In particular, the matrices \( F_l \) can be indefinite. Some methods have been proposed to cope with non-convex QCQPs, among which the semi-definite relaxation (SDR) approach is most popular [31], [32]. Unfortunately, SDR fails to reach a feasible solution in most of the cases with indefinite matrices [33]. Therefore, in previous works, successive convex approximation (SCA) algorithms have been proposed [34], [33]. Typically, these iterative algorithms are utilized in order to convert non-convex constraints into convex constraints by approximating the non-convex part and using the solution from the previous iteration. We apply the recently proposed Feasible Point Pursuit (FPP)-SCA algorithm described in [33] for solving (10). In each iteration of the algorithm, a convex second-order cone program (SOCP) results, which is solved via SDR. In case of convergence, the solution may not always be feasible (unsuccessful FPP), however, the probability of failure is very low with this algorithm (less than 7.2% in [33] and < 1% for the simulated scenarios in this work).

The resulting beamforming vector is then used in the second subproblem as additional constraint for the design of the optimal scheduling. The result of the optimization is vector \( V \), which contains the beamforming voltages of all pairs and SNTs. These voltages not only provide the optimal beamforming pattern for each pair or single node \( x \), but also contain a scaling factor \( \sqrt{\alpha_x} \) according to (9), which indicates the overall energy to be transmitted by this pair in order to obtain the solution of the first subproblem.

\section*{D. Scheduling}

The second subproblem is related to the question, which nodes and node pairs should transmit in particular time slots, such that the second and third constraints from (8) are not violated at any time. Hence, we provide a transmission policy, which guarantees that only the nodes with enough energy transmit using their optimum beamforming vectors, and that the energy is not transmitted to the nodes with already charged batteries.

We propose the following strategy for the scheduling and scaling of the transmit vectors. In each time slot, a scaled version of the optimal beamforming vector is transmitted. The optimal beamforming vector for each SNT and each node pair is obtained from the solution of the first subproblem, whereas the optimal scaling factor is determined, such that the problem constraints are not violated. This scaling factor is related to the maximum portion of the total energy the respective node or node pair has to transmit according to the solution of the first subproblem. In each time slot, the node/node pair with the largest portion is selected for transmission. For this, the scaling factors \( h_{l,1}(m) \) and \( h_{x,2}(m) \) are introduced for SNTs and node pairs, respectively. Index 1 in \( h_{l,1}(m) \) denotes the number of simultaneous transmissions per time slot. This additional index is introduced in order to distinguish between scaling factors of different tuples, as discussed in Section III-E. This strategy has been chosen, since it guarantees that all problem constraints are satisfied. A distinct advantage is that the largest portion of the total transmit energy of the particular node/node pair is transmitted in each time slot. Intuitively, this leads to a faster charging, since the energy is transmitted in large portions instead of small ones. Hence, less time slots are needed to charge the whole network. At first, we consider only the single node transmissions. Assume that node \( l \) transmits in time slot \( m \). Since no overlap with the signals from other nodes can be expected, the phase of the complex voltage \( e^7 V_1 \) is irrelevant. However, the amount of energy to be transmitted by node \( l \) is restricted by the following events, that can occur during transmission:

1) the battery of node \( l \) is depleted \( \Rightarrow \) not enough energy for further transmissions (constraint 2) from (8) is active;

2) the total amount of energy transmitted by \( l \) over all time slots reaches \( |e^7 V_1|^2 Z_{in}^{-1} \) (optimality criterion according to the first subproblem, as mentioned at the end of Section III-C);

3) the battery of node \( p \neq l \) gets fully charged \( \Rightarrow \) no more energy can be received by node \( p \) (constraint 3) from (8) is active).

Each of these events means violation of the constraints and must be avoided. Therefore, the transmit
energy of node $l$ should be less or equal the transmit energy, for which these events become active. Hence, the maximum energy to be transmitted in time slot $m$ is upper bounded by the minimum energy, for which one of the three events becomes active. The corresponding factor $h_{l,1}(m)$ can be determined via

$$h_{l,1}(m) = \min\{A_l(m), B_l(m), C_l(m)\},$$

where $A_l(m)$ stands for the normalized available energy at node $l$ (event 1 occurs), $B_l(m)$ represents the normalized maximum energy to be transmitted by node $l$ according to the first subproblem (event 2 occurs), and $C_l(m)$ denotes the normalized minimum transmit energy needed until the battery of any other node gets fully charged (event 3 occurs). These values are given by

$$A_l(m) = \sum_{n=1}^{m-1} \frac{\mathbf{U}_l^H(n) \mathbf{K}_l \mathbf{U}_l(n)}{|\mathbf{e}_l^T \mathbf{V}_1|^2} Z_{in}^{-1},$$

$$B_l(m) = 1 - \sum_{n=1}^{m-1} \frac{\mathbf{U}_l^H(n) \mathbf{G}_l \mathbf{U}_l(n)}{|\mathbf{e}_l^T \mathbf{V}_1|^2} Z_{in}^{-1},$$

$$C_l(m) = \min_{p \neq l} \left\{ b_p - \sum_{n=1}^{m-1} \frac{\mathbf{U}_l^H(n) \mathbf{K}_p \mathbf{U}_l(n)}{|\mathbf{e}_l^T \mathbf{V}_1|^2} + \sum_{n=1}^{m-1} \frac{\mathbf{U}_l^H(n) \mathbf{G}_p \mathbf{U}_l(n)}{|\mathbf{e}_l^T \mathbf{V}_1|^2} Z_{in}^{-1} \right\}.$$  

Here, $A_l(m)$ results from the difference between the total received energy at node $l$ and its transmitted energy before the time slot $m$. The result is normalized by the total energy $\frac{|\mathbf{e}_l^T \mathbf{V}_1|^2}{Z_{in}}$ to be transmitted by this node. $B_l(m)$ is calculated by subtracting the already transmitted energy of node $l$ from the total energy to be transmitted by this node and normalizing by $\frac{|\mathbf{e}_l^T \mathbf{V}_1|^2}{Z_{in}}$. Furthermore, in order to calculate $C_l(m)$, the remaining energy to be received by node $p$ is also normalized by $\frac{|\mathbf{e}_l^T \mathbf{V}_1|^2}{Z_{in}}$ and scaled by the factor $\frac{\mathbf{e}_l^T \mathbf{G}_l \mathbf{e}_l}{\mathbf{e}_l^T \mathbf{K}_l \mathbf{e}_l}$, which represents the inverse charging efficiency for the node $p$ using node $l$. The resulting scaling factor $h_{l,1}(m)$ is used to determine the optimal transmit voltage in time slot $m$, if node $l$ is selected for transmission as SNT. This voltage is given by $\mathbf{e}_l^T \mathbf{V}_1/\sqrt{h_{l,1}(m)}$.

Similarly, for the node pairs, consider the entries of vector $\mathbf{V}$ that correspond to the pair $x$ with nodes $\{s_1(x), s_2(x)\}$. The respective voltage vector is given by $\mathbf{U}_x = \mathbf{U}_x \sqrt{h_{x,1}(m)}$ of this beamforming solution is transmitted. $h_{x,1}(m)$ is selected similarly to (13) via

$$h_{x,2}(m) = \min\{A_x(m), B_x(m), C_x(m)\},$$

where $A_x(m)$, $B_x(m)$, and $C_x(m)$ are defined for pair $x$ as

$$A_x(m) = \min_{i \in \{1, 2\}} \left\{ \sum_{n=1, e_{s_1(x)}(U(n))=0}^{m-1} \frac{\mathbf{U}_x^H(n) \mathbf{K}_{s_i(x)} \mathbf{U}_x(n)}{|\mathbf{e}_{s_i(x)}^T \mathbf{U}_x|^2} Z_{in}^{-1} - \sum_{n=1, e_{s_1(x)}(U(n))=0}^{m-1} \frac{\mathbf{U}_x^H(n) \mathbf{G}_{s_i(x)} \mathbf{U}_x(n)}{|\mathbf{e}_{s_1(x)}^T \mathbf{U}_x|^2} Z_{in}^{-1} \right\},$$

$$B_x(m) = 1 - \sum_{n=1, e_{s_1(x)}(U(n))=0}^{m-1} \frac{\mathbf{U}_x^H(n) \mathbf{G}_{s_1(x)} \mathbf{U}_x(n)}{|\mathbf{e}_{s_1(x)}^T \mathbf{U}_x|^2} Z_{in}^{-1},$$

$$C_x(m) = \min_{p \neq (s_1(x), s_2(x))} \left\{ b_p - \sum_{n=1, e_{s_1(x)}(U(n))=0}^{m-1} \frac{\mathbf{U}_x^H(n) \mathbf{K}_p \mathbf{U}_x(n)}{|\mathbf{e}_{s_1(x)}^T \mathbf{U}_x|^2} + \sum_{n=1, e_{s_1(x)}(U(n))=0}^{m-1} \frac{\mathbf{U}_x^H(n) \mathbf{G}_p \mathbf{U}_x(n)}{|\mathbf{e}_{s_1(x)}^T \mathbf{U}_x|^2} Z_{in}^{-1} \right\}.$$  

For $A_x(m)$, the available energy at nodes $s_1(x)$ and $s_2(x)$ is normalized by the respective total transmit energies. In order to determine which node will be depleted first, the resulting energy ratios are compared and the minimum value is taken. Obviously, the relative depletion rate is equal for both nodes of the same pair, since the beamforming vector $\mathbf{U}_x$ remains unchanged in all time slots, in which pair $x$ is scheduled for transmission. Hence, $B_x(m)$ can be calculated using node $s_1(x)$ or $s_2(x)$ by subtracting the energy, which has already been transmitted by node $s_1(x)$ or $s_2(x)$, respectively, via pair $x$ from the total energy to be transmitted by the respective node in pair $x$. The result is normalized by the respective total transmit energy.
As mentioned earlier, the node/node pair with the largest scaling factor by node pair \( x \) is given by \( \frac{U_x H G_{s_{x_i}} U_x}{U_x H K_x U_x} \). We exploit the definition of matrix \( G_l \) from (7), which yields \( U_x H G_{s_{x_i}} U_x = U_x T_{s_{x_i}} U_x Z_{i-1}^{-1} \). Hence, after the normalization of the inverse charging efficiency by \( U_x T_{s_{x_i}} U_x Z_{i-1}^{-1} \), a scaling factor \( \frac{1}{U_x H K_x U_x} \) remains. As mentioned earlier, the node/node pair with the largest scaling factor \( h_{i,1} \) (or \( h_{i,2} \)) is selected for transmission. In order to illustrate this strategy, we give an example for charging a network with four nodes (including master node) and using only SNTs (\( N_{\text{simultaneous}} = 1 \) is selected for better comprehension), see Fig. 3. In the beginning of time slot 1, only the master node (node 1) has enough energy to transmit. \( A_{2}(1) = 0 \), \( A_{3}(1) = 0 \), and \( A_{4}(1) = 0 \) holds, such that \( h_{2,1}(1) = 0 \), \( h_{3,1}(1) = 0 \), and \( h_{4,1}(1) = 0 \) result, respectively. The transmit energy is chosen via the scaling factor \( h_{i,1}(1) = 0.6 \), such that one of the nodes gets fully charged. In the beginning of the second time slot, all nodes have enough energy to start a transmission. However, if node 1, 3, or 4 would transmit, node 2 may not be able to receive this energy, since its battery is already full. Hence, \( C_{1}(2) = 0 \), \( C_{3}(2) = 0 \), and \( C_{4}(2) = 0 \) holds. This yields \( h_{1,1}(2) = 0 \), \( h_{3,1}(2) = 0 \), and \( h_{4,1}(2) = 0 \), respectively. Then, node 2 is selected and transmits with \( h_{2,1}(2) = 1 \). This means, that the total energy to be transmitted by this node is sent at once in time slot 2 and this node will not transmit anymore in any consecutive time slots. In the third time slot, \( h_{1,1}(3) = 0.2 \), which is lower than \( h_{3,1}(3) = 1 \), such that node 3 is selected. Here, the total energy to be transmitted by node 3 is completely consumed during time slot 3. Hence, node 3 will not transmit in any consecutive time slots anymore. Finally, in the fourth time slot, \( B_{2}(4) = 0 \), \( B_{3}(4) = 0 \), and \( B_{4}(4) = 0 \) holds, since nodes 2, 3, and 4 have reached their total transmit energy obtained from the first subproblem. Hence, the master node transmits with \( h_{1,1}(4) = 0.4 \) and all nodes get fully charged, such that the whole charging procedure only takes four time slots. After that, the master node has consumed the total transmit energy suggested by the solution of the first subproblem, as can be seen from the sum \( h_{1,1}(1)+h_{1,1}(4) = 1 \). Note, that \( h_{1,1}(3) = 0.2 \) is not included in the sum, since another node has been selected in time slot 3. Interestingly, the master node is supposed to transmit in an SNT in the first and the last time slot, as can be deduced directly from the problem formulation (8). In the first time slot, the remaining nodes do not yet have any energy for transmission. In the last time slot, none of the remaining nodes is allowed to transmit in order to not violate the first constraint, see Fig. 3.

With the proposed scheduling, all constraints of (8) are satisfied and a close-to-optimum performance of the proposed solution can be expected.

E. More transmitters (\( N_{\text{simultaneous}} > 2 \))

For \( N_{\text{simultaneous}} > 2 \), both subproblems need to be extended to a more general case. This can be done by redefining the matrices and vectors that have been utilized throughout previous sections. First of all, vector \( V \) is redefined as \( V = [V_T^1, V_T^2, \ldots, V_T^{N_{\text{simultaneous}}}]^T \), where \( V_1 \) of length \( N_{\text{nodes}} \) contains the voltages for the SNTs and \( V_k, k > 1 \), of length \( \binom{N_{\text{nodes}}}{k} \) contains the voltages of the \( k \)-nodes transmissions. Moreover, we redefine the matrix \( W_{g_k,k} \) of a \( k \)-tupel with index \( x_k \) and nodes \( \{s_1(x_k), \ldots, s_k(x_k)\} \) as \( W_{x_k} = \sum_{i=1}^{k} E_{s_i(x_k), (y+(x_k-1)k+i)} \), where \( y = \sum_{i=0}^{k-1} \binom{N_{\text{nodes}}}{i} - 1 \) and \( E_{g_1,g_2} \) stands for a matrix of size \( N_{\text{nodes}} \times N_{\text{simultaneous}} \) with all zero elements except for ‘1’ at the position \((g_1, g_2)\). Similarly to (11)-(12),

\[ a^T s_{x_1} x_1^2 Z_{i-1}^{-1} \text{ or } a^T s_{x_2} x_2^2 Z_{i-1}^{-1} \] for \( C_x(m) \), we obtain a result similar to (14). The inverse charging efficiency for node \( p \) using node \( s_{x}(x), i \in \{1, 2\} \), respectively. The transmit energy is chosen via the scaling factor \( \frac{1}{U_x H K_x U_x} \).
we redefine the matrices \( D \) and \( F_l \) as follows:

\[
D = \sum_{i=1}^{N_{\text{simultan}}} \sum_{x=1}^{N_{\text{nodes}}} W_{x,i} G_1 W_{x,i}^T,
\]

\[
F_l = \sum_{i=1}^{N_{\text{simultan}}} \sum_{x=1}^{N_{\text{nodes}}} W_{x,i}^T K_l W_{x,i}^T - \sum_{i=1}^{N_{\text{simultan}}} \sum_{x=1}^{N_{\text{nodes}}} W_{x,i}^T G_l W_{x,i}^T.
\]

Using these matrices, the problem formulation in (10) can remain unchanged. Also, the proposed optimization strategy is as explained earlier. The scheduling strategy for \( N_{\text{simultan}} > 2 \) is very similar to that discussed in Section III-D. The beamforming vector \( U_{x_k} \) is identical for the same tuple \( x_k \) in all time slots occupied by this tuple. Hence, the only unknown parameter is the scaling factor \( h_{x_k,k}(m) \) in time slot \( m \). Similar to the calculations (17), we obtain

\[
h_{x_k,k}(m) = \min\{ A_{x_k}(m), B_{x_k}(m), C_{x_k}(m) \},
\]

where \( A_{x_k}(m) \), \( B_{x_k}(m) \), and \( C_{x_k}(m) \) are defined for the \( k \)-tuple \( x_k \) with nodes \( \{s_1(x_k), \ldots, s_k(x_k)\} \) as

\[
A_{x_k}(m) = \min_{i \in \{1, \ldots, k\}} \left\{ \frac{\sum_{n=1}^{m-1} U_H(n) K_{s_i(x_k)} U(n)}{e_{s_i(x_k)}^T U(n) e_{s_i(x_k)} U(n)} \right\},
\]

\[
B_{x_k}(m) = 1 - \sum_{n=1}^{m-1} \frac{\sum_{i=1}^{m-n} \sum_{s_i(x_k)} U_H(n) G_{s_i(x_k)} U(n)}{e_{s_i(x_k)}^T U(n) e_{s_i(x_k)} U(n)},
\]

\[
C_{x_k}(m) = \min_{p \neq s_i(x_k), \forall i \leq k} \left\{ b_p - \frac{\sum_{n=1}^{m-1} U_H(n) K_p U(n)}{e_{p}^T U(n) e_{p} U(n)} \right\} + \sum_{n=1}^{m-1} \frac{1}{U_H(n) K_p U(n)},
\]

Using the calculated scaling factor \( h_{x_k,k}(m) \), the resulting vector \( U_{x_k} \sqrt{h_{x_k,k}(m)} \) is transmitted. Similarly to Section III-D, all tuples are sorted according to their scaling factor \( h_{x_k,k}(m) \) and the tuple with the largest factor is selected for transmission. For a better understanding of the proposed scheduling, we provide its algorithmic description in terms of pseudo code (see Appendix).

### IV. Numerical Results

In this section, we discuss numerical results on the performance of the proposed relayed powering of the sensor nodes in access limited WUSNs. In our simulations, a set of \( N_{\text{nodes}} \) sensor nodes is randomly (uniform distribution) deployed in a square field of size \( F \times F \) for each simulated scenario. In this set, a master node is selected, which is the closest node to the lower left field corner. We utilize coils with wire radius 0.5 mm, coil radius 0.25 m, and \( N_w = 500 \) coil windings. The conductivity of soil is \( \sigma = 0.01 \text{ S/m} \) [10]. Since the permeability of soil is close to that of air, we use \( \mu = \mu_0 \) with the magnetic constant \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \). The resonance frequency is set to \( f_0 = 1 \text{ MHz} \). Moreover, for the charging mode, we assume that all batteries have the same capacity of 0.1 J [20]. In this work, we focus on minimum spanning tree-based WUSNs [8]. Furthermore, in the information transmission mode (while not being charged), we assume that each node not only transmits its own packets, but also relays the previously received data packets. Hence, the more data streams each node has to serve, the more packets need to be transmitted from this node [35]. Correspondingly, the depletion of the nodes’ batteries is not uniform in the network [36]. We assume, that the network starts the charging procedure, if one of the nodes’ batteries is completely empty. Obviously, the empty battery belongs to the node \( l_{\text{max}} \), which serves the largest number of data streams \( \max_{n} \{N_{\text{streams},n}\} \), where \( N_{\text{streams},n} \) denotes the number of data streams served by node \( n \). The amount of energy consumed by node \( l \) until the battery of node \( l_{\text{max}} \) gets completely depleted depends on the number of data streams \( N_{\text{streams},l} \) and corresponds to the required battery charge \( b_l \) given by

\[
b_l = \frac{\max_{n} \{N_{\text{streams},n}\}}{0.1 \text{ J}}.
\]

In the following, we provide a cumulative distribution of energy efficiency for 1000 realizations for each of the considered scenarios and compare the baseline scheme (only master node transmits) with the proposed solution for \( N_{\text{simultan}} = \{1, 2, 3\} \). Note, that the energy efficiency is defined as a ratio of
∑_i b_i \over \text{total transmitted energy from the master node, as mentioned earlier. Here, the solution for } N_{\text{simult}} = 1 \text{ can be easily obtained either by setting all entries of the respective matrices } W_x \text{ to zero for all pairs and solving the problem as described in Section III, or by formulating a simplified optimization problem with reduced complexity (typically a linear program), which can be solved via the Simplex algorithm.}

At first, we show the performance for charging 8 nodes (including master node) for } F = 35 \text{ m, see Fig. 4. We observe that the proposed solution outperforms the baseline scheme on average by 30\%, 115\%, and 140\% with } N_{\text{simult}} = \{1, 2, 3\} \text{, respectively. However, the average energy efficiency for the optimized solution is below } 10^{-4} \text{ even with three nodes transmitting simultaneously. This is due to the harsh transmission conditions and a large number of hops that separate the master node from the leaf nodes. Also, we observe that the efficiency gain using } N_{\text{simult}} = 3 \text{ compared to } N_{\text{simult}} = 2 \text{ is much smaller than that for using } N_{\text{simult}} = 2 \text{ compared to } N_{\text{simult}} = 1 \text{. From this, we deduce that the achievable efficiency using multiple nodes in simultaneous transmissions converges fast. Since the efficiency gain is obtained at the price of a higher system complexity (both for computation and implementation of scheduling), the choice of } N_{\text{simult}} \text{ for the final system design represents a trade-off between complexity and energy efficiency. For this particular scenario, } N_{\text{simult}} = 3 \text{ seems to be reasonable, due to a significant efficiency improvement of 22\% on average compare to } N_{\text{simult}} = 2 \text{. For a smaller network with only 5 nodes and } F = 20 \text{ m, the results are depicted in Fig. 5. The proposed solution outperforms the baseline scheme by 20\%, 59\%, and 63\% on average for the respective values } N_{\text{simult}} = \{1, 2, 3\}. \text{ This efficiency gain is somewhat less than with 8 nodes, which is due to the shorter transmission distances between the master node and the leaf nodes, such that all nodes can receive more energy directly from the master node. In addition, a small network with only 5 nodes can be charged much more efficiently than a network with 8 nodes. Here, the average energy efficiency is 0.26\% for } N_{\text{simult}} = 3 \text{, which is } \approx 40 \text{ times larger than with 8 nodes. From this, we deduce that a good strategy (if feasible) could be to partition the network in small clusters, which can be charged independently. This may dramatically improve the performance since the power signals would need less hops on average to reach the target nodes. Furthermore, in this scenario, we observe an even faster convergence of the energy efficiency, such that the efficiency gain using } N_{\text{simult}} = 3 \text{ compared to } N_{\text{simult}} = 2 \text{ is negligibly small. Hence, } N_{\text{simult}} = 2 \text{ seems a better choice than } N_{\text{simult}} = 3 \text{ due to a much lower complexity.}

In addition, we investigate a scenario which is even more realistic for the WUSNs in mines and tunnels. For this, we assume that the directions of the network links correspond to the directions of the...
tunnels. Therefore, if a particular link is not part of the spanning tree (which is the assumed topology in this work), the corresponding devices must be separated by soil. In such cases, the assumption that the coils are deployed at the ground surface is not valid. Hence, we use $G_{k,l}$ from (3) instead of (2). The resulting energy efficiency is depicted in Fig. 6. Due to much weaker coupling, it is hardly possible for the master node to charge the leaf nodes via direct coupling. In principle, the performance of the baseline scheme relies mostly on the passive relaying of magnetic field (also known as MI waveguide [37]), which is, however, very weak [9]. This explains the remarkable average efficiency gain of 134%, 245%, and 269% for the proposed solution using $N_{\text{simultan}} = \{1,2,3\}$, respectively. Even with $N_{\text{simultan}} = 2$, we obtain more than 109% gain in 50% of considered realizations, and a peak efficiency gain of 5733% compared to the baseline scheme. Unfortunately, the average energy efficiency ($\approx 6.6 \cdot 10^{-4}$) further decreases compared to Fig. 5 (approximately by factor 3.9), which is because less nodes can be charged simultaneously. Hence, more hops are needed in order to charge the leaf nodes. Correspondingly, the losses per hop accumulate and decrease the overall charging efficiency.

V. CONCLUSION

In this work, a novel solution for the wireless power transfer in access limited WUSNs is presented. The problem of charging all sensor nodes in the most efficient way can be typically solved via multihop energy transfer as proposed in the previous works. In our approach, we assume that several sensor nodes can synchronize their transmissions for the charging procedure in order to maximize the energy transfer efficiency in each time slot. Of course, the resulting beamforming pattern varies from time slot to time slot, since it depends on the time-varying state of energy levels of all sensor nodes. Hence, a more general optimization problem results, where a multinode scheduling needs to be established, such that the optimal beamforming pattern is obtained in all consecutive time slots. This optimization problem is shown to be non-linear and non-convex in general. Hence, the optimum solution cannot be determined analytically. In the proposed approach, the optimization problem is split in two subproblems, and each subproblem can be solved independently. The first subproblem turns out to be a non-convex QCQP problem, which is solved using a recently proposed FPP-SCA algorithm. The proposed suboptimal solution for the second subproblem represents a power transfer policy, where the best beamforming vector is selected and optimally scaled in each time slot. Significant energy efficiency gains compared to the baseline scheme are observed even for small networks with few sensor nodes. Furthermore, MI-WUSNs in tunnels have been investigated, where the signal transmission between two devices is exposed a much larger path loss, if these devices are separated by soil instead of a tunnel. In this constellation, even larger efficiency gains can be achieved by the proposed solution. However, the expected efficiency for MI-WUSNs in tunnels is heavily reduced due to a lower number of nodes that can be charged simultaneously. In addition, a trade-off between complexity of the system and the energy efficiency has been discussed. It has been shown, that the achievable energy efficiency converges very fast for an increasing number of simultaneous transmissions, such that at most two nodes transmitting in the same time slot are enough in most of the considered scenarios. Correspondingly, the efficiency gains using three or more nodes transmitting in the same time slot are fairly negligible, and such transmissions are non-preferable due to a tremendously higher complexity.
APPENDIX

A. Proof of Theorem 1

Assume that the beamforming vectors are optimal in terms of (8) for all time slots. Furthermore, we assume that pair \( x \) with nodes \( \{s_1(x), s_2(x)\} \) is selected for transmission in time slots \( n_1 \) and \( n_2 \). As mentioned earlier, all energy available for transmission of pair \( x \) stems from the master node and can be expressed as a partial of its total transmitted energy. In the following, we describe the properties of the optimal beamforming solution for time slots \( n_1 \) and \( n_2 \). The corresponding beamforming vectors \( \mathbf{U}(n_1) \) and \( \mathbf{U}(n_2) \) can be viewed as the optimal solution for the following (relaxed) optimization problem ((8) without constraints (2) and (3)):

\[
\min_{\mathbf{U}(n_1), \mathbf{U}(n_2)} \sum_{n=1}^{N_t} \mathbf{U}(n) \mathbf{G}_1 \mathbf{U}(n), \quad (27)
\]

s.t.: \( \mathbf{U}(n_1) \mathbf{K}_1 \mathbf{U}(n_1) + \mathbf{U}(n_2) \mathbf{K}_1 \mathbf{U}(n_2) \geq b_t - \sum_{n=1, n \neq n_1, n \neq n_2} \mathbf{U}(n) \mathbf{G}_1 \mathbf{U}(n), \quad (29)\)

\[
\alpha \mathbf{U}(n_1) \mathbf{G}_1 \mathbf{U}(n_1) - \mathbf{U}(n_2) \mathbf{G}_1 \mathbf{U}(n_2) \geq b_t - \sum_{n=1, n \neq n_1, n \neq n_2} \mathbf{U}(n) \mathbf{G}_1 \mathbf{U}(n), \quad (29)\)

where (28) represents the energy to be transmitted by the pair \( x \) in time slots \( n_1 \) and \( n_2 \) described as the total transmit energy from the master node scaled by a constant factor \( 1 > t > 0 \). This representation is valid, since we assume that the available energy in all nodes of the network needs to be first received from the master node prior to transmission. By inserting (28) in the cost function of (27), we obtain

\[
\min_{\mathbf{U}(n_1), \mathbf{U}(n_2)} \frac{1}{t} \left( \mathbf{U}(n_1) \left( \mathbf{G}_{s_1(x)} + \mathbf{G}_{s_2(x)} \right) \mathbf{U}(n_1) + \mathbf{U}(n_2) \left( \mathbf{G}_{s_1(x)} + \mathbf{G}_{s_2(x)} \right) \mathbf{U}(n_2) \right) \quad (29)
\]

s.t.: \( \mathbf{U}(n_1) \mathbf{K}_1 \mathbf{U}(n_1) + \mathbf{U}(n_2) \mathbf{K}_1 \mathbf{U}(n_2) \geq q_t, \quad \forall l \notin \{s_1(x), s_2(x)\}, \)

\[
\mathbf{U}(n_1) \mathbf{G}_1 \mathbf{U}(n_1) - \mathbf{U}(n_2) \mathbf{G}_1 \mathbf{U}(n_2) \geq q_t, \quad \forall l \notin \{s_1(x), s_2(x)\}, \quad (29)
\]

with implicit definition of \( q_t \). Then, we substitute \( \mathbf{U}(n_1) = T_1 \mathbf{V}_1 \) and \( \mathbf{U}(n_2) = T_2 \mathbf{V}_2 \) with \( T_1^H T_1 = 1, T_2^H T_2 = 1 \), and real positive constants \( \alpha \) and \( \beta \). Furthermore, \( T_1^H \left( \mathbf{G}_{s_1(x)} + \mathbf{G}_{s_2(x)} \right) T_1 = \frac{\mathbf{G}_1 - \mathbf{G}_2}{Z_{in}} \) holds using the definition of \( \mathbf{G}_{s_1(x)} \) and \( \mathbf{G}_{s_2(x)} \) in (7). Hence, the problem (29) can be rewritten into

\[
\min_{T_1, T_2, \alpha, \beta} \frac{(\alpha + \beta)}{Z_{in} t} \quad (30)
\]

s.t.: \( \alpha T_1^H \mathbf{K}_1 T_1 + \beta T_2^H \mathbf{K}_2 T_2 \geq q_t, \quad \forall l \notin \{s_1(x), s_2(x)\}, \)

\[
-\alpha T_1^H \mathbf{G}_1 T_1 - \beta T_2^H \mathbf{G}_2 T_2 \geq q_t, \quad \forall l \notin \{s_1(x), s_2(x)\}. \quad (32)
\]

Typically, the optimal solution for this problem fulfills at least one constraint with equality. Otherwise, the beamforming vector can be scaled, such that less energy is consumed at the transmitter, which proves that the obtained solution was suboptimal. Hence, for the optimal solution, we can assume that all constraints are fulfilled with inequality except for the constraint \( m \), which is fulfilled with equality. Furthermore, \( m \notin \{s_1(x), s_2(x)\} \) holds, since otherwise the node pair \( x \) would finish the transmission before some of the remaining nodes get fully charged. Hence, constraint \( m \) is obtained from (31). Using this constraint, the relation between \( \alpha \) and \( \beta \) can be established:

\[
\alpha = \frac{q_m - \beta T_2^H \mathbf{K}_m T_2}{T_1^H \mathbf{K}_m T_1}, \quad \beta = \frac{q_m}{T_1^H \mathbf{K}_m T_1} - \frac{T_2^H \mathbf{K}_m T_2}{T_1^H \mathbf{K}_m T_1}. \quad (33)
\]

Then, for the cost function in (30) we obtain

\[
(\alpha + \beta) \frac{1}{Z_{in} t} = \frac{q_m}{T_1^H \mathbf{K}_m T_1} + \beta \left( 1 - \frac{T_2^H \mathbf{K}_m T_2}{T_1^H \mathbf{K}_m T_1} \right). \quad (34)
\]
Similarly, we obtain

\[(\alpha + \beta) \frac{1}{Z_{int}} = \frac{q_m}{Z_{int}} + \alpha \left(1 - \frac{T_1^H K_m T_1}{T_2^H K_m T_2}\right),\]

if the equality constraint is used to express \(\beta\) as a function of \(\alpha\). Obviously, if \(T_1 = T_2\), the cost function can reach the value \(\frac{q_m}{Z_{int}}\). Then, the optimal solution for \(T_1\) is given by the eigenvector that corresponds to the maximum eigenvalue of \(K_m\). If \(T_1 \neq T_2\), then three cases are possible:

1) Assume \(T_1^H K_m T_1 > T_2^H K_m T_2\). Then, \(\frac{\alpha + \beta}{Z_{int}} > \frac{q_m}{Z_{int}}\) results from (34), which is worse than with \(T_1 = T_2\).

2) Similarly, for \(T_1^H K_m T_1 < T_2^H K_m T_2\), we obtain \(\frac{\alpha + \beta}{Z_{int}} < \frac{q_m}{Z_{int}}\) from (35), which is suboptimal as well.

3) For \(T_1^H K_m T_1 = T_2^H K_m T_2\), the optimal vectors \(T_1\) and \(T_2\) correspond to the two eigenvectors of \(K_m\) with the largest eigenvalues. These eigenvalues should be equal in order to guarantee \(T_1^H K_m T_1 = T_2^H K_m T_2\). However, it can be shown that \(K_m\) has only one eigenvector with a positive eigenvalue. Hence, \(T_1^H K_m T_1 = T_2^H K_m T_2\) only holds for \(T_1 = T_2\).

Hence, the optimal solution is obtained for the case \(T_1 = T_2\). Then, substitution of variables yields \(U(n_1) = \frac{\sqrt{\alpha}}{\sqrt{\beta}} U(n_2) = c \cdot U(n_2)\) with \(c = \frac{\sqrt{\alpha}}{\sqrt{\beta}}\), q.e.d.

**B. Scheduling algorithm for \(N_{simultan} \geq 2\)**

**Algorithm 1**

1: Input: \(U_{x_k}, \forall k, x_k\) from FPP-SCA;
2: \(m = 0\);
3: while constraint 1 from (8) not fulfilled for \(N_{ts} = m\) do
4: \(m = m + 1\);
5: \(h_{max}(m) = 0\), \(U(m) = e_1\);
6: for \(k = 1\) to \(N_{simultan}\) do
7: for \(x_k = 1\) to \(k\) do
8: calculate \(A_{x_k}(m), B_{x_k}(m),\) and \(C_{x_k}(m)\) using (24)-(26);
9: determine \(h_{x_k,k}(m)\) using (23);
10: if \(h_{x_k,k}(m) > h_{max}(m)\) then
11: \(h_{max}(m) = h_{x_k,k}(m)\);
12: \(U(m) = U_{x_k}\sqrt{h_{x_k,k}(m)}\);
13: end if
14: end for
15: end for
16: if \(h_{max}(m) = 0\) then
17: break;
18: end if
19: end while
20: \(N_{ts} = m\);
21: Output: \(U(m), 1 \leq m \leq N_{ts}\).

**REFERENCES**


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