Occlusion-Aware Fragment-based Tracking with Spatial-Temporal Consistency

Chong Sun, Dong Wang, Huchuan Lu

Abstract—In this paper, we present a robust tracking method by exploiting a fragment-based appearance model with consideration of both temporal continuity and discontinuity information. From the perspective of probability theory, the proposed tracking algorithm can be viewed as a two-stage optimization problem. In the first stage, by adopting the estimated occlusion state as a prior, the optimal state of the tracked object can be obtained by solving an optimization problem, where the objective function is designed based on the classification score, occlusion prior and temporal continuity information. In the second stage, we propose a discriminative occlusion model, which exploits both foreground and background information to detect the possible occlusion, and also models the consistency of occlusion labels among different frames. In addition, a simple yet effective training strategy is introduced during the model training (and updating) process, with which the effects of spatial-temporal consistency are properly weighted. The proposed tracker is evaluated by using the recent benchmark dataset, on which the results demonstrate that our tracker performs favorably against other state-of-the-art tracking algorithms.

Index Terms—visual tracking, fragment-based appearance model, spatial-temporal consistency, occlusion model.

I. INTRODUCTION

The goal of visual tracking is to identify the position of a pre-specified object continuously in a given image sequence, which is very attractive in computer vision for its wide applications in numerous domains. Nowadays, it still remains a challenging task to develop a sophisticated algorithm which performs object tracking robustly and accurately due to many difficulties. These difficulties are mainly attributed to appearance changes, severe occlusion, illumination variations, background clutter and so on.

For developing a robust and effective tracking algorithm, it is very important to fully consider three critical issues: appearance model, temporal continuity and temporal discontinuity. Existing tracking algorithms [1]–[7] mainly focus on designing a robust appearance model due to its core role in the tracking problem. Recently, many fragment-based methods are exploited to enhance the appearance representation of the tracked object and therefore to improve the tracking performance (e.g., [5], [8]–[15]). Compared with the “holistic representation”-based trackers (i.e., use a single regular bounding box to describe the target), the fragment-based algorithms are more discriminative in distinguishing the tracked object from its surrounding background. Adam et al. [8] divide the object region into several fragments, and then locate the target’s position by fusing the voting maps of these fragments. Although this method is robust to occlusion, it is less effective in handling pose change and background clutter as spatial information among different fragments are not fully exploited (especially the spatial configuration of different fragments is fixed). To address this issue, a novel fragment-based tracking method [15] is proposed by introducing spatial latent variables within a structured support vector machine (SVM) framework. This work adopts a spatial tree structure to describe the appearance of the tracked object, in which an object is represented by a holistic fragment and some sub-fragments. The spatial configuration of different fragments is fully considered by introducing the spatial relation vectors into the model training (and updating) process. Quite recently, Liu et al. [16] exploit multiple patch-based correlation filters to obtain several response maps, and propose an adaptive weighting method to fuse these response maps for object tracking.

Temporal continuity (usually called motion model) aims to depict the state transition of the tracked object over time, which is a very important characteristic in visual tracking. In recent “particle filter”-based trackers (such as [2], [17]–[21]), they usually assume that the motion of the tracked object between two consecutive frames follows a Gaussian distribution. This assumption is too rough as it does not take motion estimation into account. Thus it limits the tracking performance especially when some large or complex motions occur. Although the motion estimation is well studied in video segmentation methods (e.g., [22], [23]), it has not been paid much attention to in the tracking field. Bai et al. [22] introduce temporal continuity into a shape model, and estimate the motion of the shapes between two consecutive frames by computing an affine transform. In [24], temporal consistent segmentations of moving objects are achieved by clustering the point trajectories which have different survival time. For visual tracking, papers [25]–[27] cast motion estimation as a linear assignment problem based on a set of short tracklets. Supančič and Ramanan [28] exploit the optical flow estimation across frames, based on which a thresholded motion model is designed to describe the motion of the tracked object. This motion model penalizes motions that violate the optical flow estimation, and therefore makes the tracker more effective in handling many complex challenges (such as out-of-plane rotation, deformation and so on). However, this thresholded
A two stage optimization manner is developed. In each stage, the recursive energy function is derived to model different components. In our tracking framework, we firstly adopt a joint distribution to depict both locations and occlusion states. Then, we exploit a two stage optimization strategy to infer the optimal locations and best occlusion states separately rather than solve them jointly. For one thing, the model parameters (i.e., classification coefficients) in the joint distribution are updated moderately to avoid model degradation and tracking drift. For another, the occlusion model exploits more recent samples, and is updated faster to account for state changes of occlusion. These two models enhance each other in the tracking process.

Promising tracking performance is achieved in the CVPR2013 benchmark [32]. We conduct extensive experiments to compare our tracker with numerous state-of-the-art methods on the recent benchmark dataset [32]. It can be seen from experimental results that the proposed tracker performs favorably against the compared state-of-the-art trackers.

**II. BAYESIAN INFERENCE FRAMEWORK**

In this work, we present the proposed tracking framework from the perspective of probability theory. Here, we use \( X_{1:t} = \{X_1, X_2, \ldots, X_t\} \) to denote the state set of the tracked object till the \( t \)-th frame, and utilize \( Y_{1:t} = \{Y_1, Y_2, \ldots, Y_t\} \) to denote the corresponding observation set. We use a spatial tree structure to represent the tracked object, the state of the \( t \)-th frame can be represented as \( X_t = \{x_{t,0}^1, x_{t,1}^1, \ldots, x_{t,M}^t\} \), where \( x_{t,0}^j \) denotes the state of the holistic template (i.e., fragment 0) and \( x_{t,j}^j \) stands for the state of the \( j \)-th sub-fragment. The observation \( Y_t \) can be also represented as a collection of all fragments \( Y_t = \{y_{t,0}^1, y_{t,1}^1, \ldots, y_{t,M}^t\} \), in which \( y_{t,i} \) is the observation related with the state \( x_{t,i}^i \). In this way, the state \( x_{t,i}^i \) includes two components, i.e., \( x_{t,i}^i = \{b_{t,i}, o_{t,i}^i\} \), where \( b_{t,i} = [x_{t,i}^i, y_{t,i}^i] \) is the bounding box (with center \( (x_{t,i}^i, y_{t,i}^i) \)) of the \( j \)-th fragment and \( o_{t,i}^i \) is the occlusion label to indicate whether this fragment is occluded (1 for occlusion, and 0 for non-occlusion). Thus, the state set can be rewritten as \( X_{1:t} = \{B_{1:t}, o_{1:t}\} \), where \( B_{1:t} = \{B_1, B_2, \ldots, B_t\} \) denotes the bounding box set \( (B_k = \{b_{k,1}, b_{k,2}, \ldots, b_{k,M}\}, k = 1, \ldots, l) \) and \( o_{1:t} = \{o_1, o_2, \ldots, o_t\} \) stands for the occlusion state set \( (o_k = [o_{k,0}, o_{k,1}, \ldots, o_{k,M}], k = 1, \ldots, l) \). The candidates of \( B_t \) can be efficiently obtained via the sliding window search strategy. More details about the definitions of our notations are in Table I.
TABLE I

BASIC NOTATIONS FOR THE PROPOSED METHOD.

<table>
<thead>
<tr>
<th>Notations</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b^t_j$</td>
<td>Central position of the $j$-th fragment in frame $t$</td>
</tr>
<tr>
<td>$B_t$</td>
<td>$B_t ={b^t_1, b^t_2, ..., b^t_L}$</td>
</tr>
<tr>
<td>$o^t_j$</td>
<td>Occlusion state of the $j$-th fragment in frame $t$</td>
</tr>
<tr>
<td>$x^t_j$</td>
<td>$x^t_j = {o^t_j}$</td>
</tr>
<tr>
<td>$o_t$</td>
<td>$o_t = [o^t_1, o^t_2, ..., o^t_L]$</td>
</tr>
<tr>
<td>$y^t_j$</td>
<td>Observation related to $b^t_j$</td>
</tr>
<tr>
<td>$d^t_{ij}$</td>
<td>Spatial configurations between fragment 0 and $j$</td>
</tr>
<tr>
<td>$u^t_j$</td>
<td>Linear weight vector for the $j$-th fragment</td>
</tr>
<tr>
<td>$v^t_j$</td>
<td>Linear weight vector for the spatial relation vector</td>
</tr>
<tr>
<td>$c^t_j$</td>
<td>Linear weight vector for temporal relation vector</td>
</tr>
<tr>
<td>$p^t_j$</td>
<td>Displacement vector estimated via optical flow</td>
</tr>
<tr>
<td>$S_t(B_t)$</td>
<td>Observation score for bounding box set $B_t$</td>
</tr>
<tr>
<td>$f_{t_1}(b^t_j)$</td>
<td>Feature score for bounding box $b^t_j$</td>
</tr>
<tr>
<td>$g_{t_1}(b^t_j, b^t_j)$</td>
<td>Spatial score between bounding boxes $b^t_i$ and $b^t_j$</td>
</tr>
<tr>
<td>$s_{t_1}(b^t_j, b^t_j)$</td>
<td>Spatial relation vector between fragment 0 and fragment $j$</td>
</tr>
<tr>
<td>$S_{t_1}(B_{t_1})$</td>
<td>Temporal consistency score</td>
</tr>
<tr>
<td>$S_{t_1}(B_{t-1})$</td>
<td>Observation score (similar with $S_t(B_t)$)</td>
</tr>
<tr>
<td>$S_{t_1}(B_{t_1}, B_{t_1})$</td>
<td>Transfer score between $B_{t_1}$ and $B_t$</td>
</tr>
<tr>
<td>$s_{t_1}(b^t_j, b^t_{j-1}, p^t_{j})$</td>
<td>Temporal relation vector between frames 1 and $t-1$</td>
</tr>
<tr>
<td>$g_{t_1}(b^t_j, b^t_{j-1})$</td>
<td>Transfer score between two frames for the $j$-th fragment</td>
</tr>
<tr>
<td>$e_{t_1}(o^t_j)$</td>
<td>Occlusion score for the $j$-th fragment</td>
</tr>
<tr>
<td>$h_{t_1}(o^t_j, o^t_{j+1})$</td>
<td>Probability that the $j$-th fragment is occluded</td>
</tr>
<tr>
<td>$S_{t_1}(o^t_j, o^t_{j+1})$</td>
<td>Temporal consistency of occlusion labels</td>
</tr>
</tbody>
</table>

The tracking process can be conducted via the maximum a posterior (MAP) estimation,

$$\hat{X}_{1:t} = \arg \max_{X_{1:t}} p(X_{1:t}|Y_{1:t}).$$

(1)

The posterior $p(X_{1:t}|Y_{1:t})$ can be rewritten as,

$$p(X_{1:t}|Y_{1:t}) = p(B_{1:t}|Y_{1:t}o_{1:t-1})$$

$$= p(B_{1:t}|Y_{1:t}o_{1:t})p(o_{1:t}|Y_{1:t})$$

$$= p(o_{1:t}|Y_{1:t})p(B_{1:t}|Y_{1:t}).$$

(2)

Thus, the state estimation for $X_{1:t}$ can be estimated iteratively by the following two steps: (1) given $o_{1:t}$, the optimal $B_{1:t}$ can be obtained by $B_{1:t} = \arg \max_{B_{1:t}} p(B_{1:t}|Y_{1:t}o_{1:t})$; (2) given $B_{1:t}$, the optimal $o_{1:t}$ can be obtained by $o_{1:t} = \arg \max_{o_{1:t}} p(o_{1:t}|Y_{1:t}B_{1:t})$. However, this iterative manner makes the tracking process be intractable.

By assuming that the variation of the occlusion labels between two consecutive frames is very small, we exploit the following two-stage estimation process for the $t$-th frame.

- **The first stage: computing the bounding boxes $\hat{B}_{1:t}$**
  
  First, suppose that we have obtained the occlusion label set $o_{1:t-1} = \{o_1, o_2, ..., o_{t-1}\}$ in frame $t$, and then we estimate the optimal bounding boxes $\hat{B}_{1:t}$ based on the posterior probability $p(B_{1:t}|Y_{1:t}o_{1:t-1})$. The posterior probability $p(B_{1:t}|Y_{1:t}o_{1:t-1})$ can be expanded as (see Appendix A for more details)

$$p(B_{1:t}|Y_{1:t}o_{1:t-1}) = p(Y_{1:t}|B_{1:t}o_{1:t-1})p(B_{1:t}|Y_{1:t}o_{1:t-1})$$

$$= p(Y_{1:t}|B_{1:t}o_{1:t-1})p(B_{1:t}|Y_{1:t-1}o_{1:t-1}).$$

(3)

We introduce a score function $S(B_{1:t})$ (equation (4)) that converts the MAP rule $\max_{B_{1:t}} p(B_{1:t}|Y_{1:t}o_{1:t-1})$ to the optimization problem $\max_{B_{1:t}} S(B_{1:t})$.

$$S(B_{1:t}) = S_A(B_t) + S_T(B_{1:t}),$$

(4)

where $S_A(B_t) = \log p(Y_{1:t}|B_t o_{1:t-1})$ is exploited to denote the score function of the appearance model and $S_T(B_{1:t}) = \log p(B_{1:t}|Y_{1:t-1}o_{1:t-1})$ stands for the score function of the temporal consistency model. The definitions and discussions of $S_A(\cdot)$ and $S_T(\cdot)$ are presented in Section III.

In the first stage, we merely use $o_{t-1}$ to approximately replace $o_t$ for estimating the bounding boxes of the target in the $t$-th frame. To alleviate error accumulation, the occlusion state $o_t$ will be corrected in the second stage as follows.

- **The second stage: inferring the occlusion labels $\hat{o}_{1:t}$**
  
  Second, after obtaining the optimal bounding boxes $\hat{B}_{1:t}$, we infer the occlusion label set by $\hat{o}_{1:t} = \arg \max_{o_{1:t}} p(o_{1:t}|Y_{1:t}B_{1:t})$. In this work, we introduce an occlusion score to model occlusions explicitly, i.e.,

$$S_o(o_{1:t}) = \log p(o_{1:t}|Y_{1:t}B_{1:t}),$$

(5)

the specific definition of which will be introduced in the next section. Thus, the optimal $\hat{o}_{1:t}$ can be obtained by $\hat{o}_{1:t} = \arg \max_{o_{1:t}} S_o(o_{1:t})$ instead.

| Fig. 1. An example for two kinds of target division. |

### III. Problem Formulation

In this section, we first introduce the appearance and temporal consistency models, which construct a spatial-temporal tree. Then we show how to perform inference in this tree structure, and obtain the bounding box centers. Based on these estimated centers, we introduce how we estimate the target scale with a simple and effective method. At last, we introduce the discriminatively learned occlusion model.

#### A. Fragment-based Appearance Model

First, we introduce a fragment-based appearance model and define a score function to depict it, i.e., $S_A(B_t) = \log p(Y_{1:t}|B_t o_{1:t-1})$. We define the score function as,

$$S_A(B_t) = \sum_{j=0}^{M} f_A(b_j^t) + \lambda \sum_{j=1}^{M} g_A(b_0^t, b_j^t),$$

(6)
where feature scores for different fragments \( f_A(b'_i) \), \( i \in \{0, ..., M\} \), are defined as

\[
f_A(b'_i) = \begin{cases} 
    \langle u'_i, y'_i \rangle & j = 0 \\
    \langle u'_i, y'_i \rangle & 0 < j \neq 0 \\
    \langle u'_i, y'_i \rangle & j = 1, \ 0 < j \\
\end{cases} \tag{7}
\]

In this term, \( y'_i \) is the observation feature of the \( j \)-th fragment (related with \( j \)-th bounding box \( b'_i \) visualized in Figure 1), and \( u'_i \) is the linear weight vector of \( j \)-th fragment (i.e., the coefficients of the linear SVM classifier in this work).

This term aims to depict the score that fragment \( j \) is in the position \( b'_i \). The score will be set as 0 if fragment \( j \) is detected occluded.

The second term in equation (6) denotes the spatial score, which models spatial relationships between different fragments (except fragment 0) and the holistic template (fragment 0), in which the basic function \( g_A(\cdot) \) is defined as

\[
g_A(b'_i, b'_j) = \langle v'_i, \phi_s(b'_i, b'_j) \rangle, \tag{8}
\]

where \( \phi_s(b'_i, b'_j) \) is the spatial relation vector between fragment 0 and fragment \( j \) that can be calculated by

\[
\begin{align*}
  dx &= x'_i - x'_j - d'_j[1] \\
  dy &= y'_i - y'_j - d'_j[2]
\end{align*}
\]

and \( v'_i \) denotes the corresponding classification coefficient vector. The vectors \( d'_j, j \in \{1, ..., M\} \), depict the spatial configurations between fragment 0 and fragment \( j \), which are online updated every frame (i.e., the relative positions of fragments are not fixed in this work).

### B. Temporal consistency model

In many recent trackers \([2, 17, 33, 34]\), motions between consecutive frames are usually assumed as a Gaussian random walk process. This assumption limits the tracking performance for two main reasons: (1) it is not very accurate to model the object motion as a Gaussian distribution (see Appendix B for more discussions); (2) information from previous frames is lost as only two consecutive frames are considered. In this paper, we address these problems by using a recursive temporal consistency model.

Recall the second term in equation (3), where the probability \( \rho(B_{t,t}\mid Y_{t,t-1}, o_{t-1}) \) can be easily decomposed as

\[
p(B_{t,t}\mid Y_{t,t-1}, o_{t-1}) \propto p(B_{t,t}\mid Y_{t,t-1}, o_{t-1}) \\ p(B_{t,t}\mid o_{t-1}) \times p(B_{t-1,t}\mid o_{t-1}) \times p(Y_{t,t-1}\mid B_{t-1,t}). \tag{10}
\]

In our work, we determine the joint distribution of bounding box set \( B_{t-1,t} \) and the observation likelihood of \( B_{t-1,t} \) with occlusion labels in previous frames (i.e., \( o_{t-1,t-2} \)). Thus equation (10) can be written as

\[
p(B_{t,t}\mid Y_{t,t-1}, o_{t-1}) \propto p(Y_{t,t-1}\mid B_{t-1,t}, o_{t-1}) \times p(B_{t,t}\mid o_{t-1}) \times p(B_{t-1,t}\mid Y_{t,t-2}, o_{t-2,t-2}). \tag{11}
\]

By taking the logarithm operator at both sides of equation (11), we can obtain the following equation,

\[
S_T(B_{1:t}) = S_T^1(B_{1:t-1}) + S_T^2(B_{1:t-1}) + S_T(B_{1:t-1}) + C_t, \tag{12}
\]

where

\[
S_T^1(B_{1:t}) = \log p(B_{1:t}\mid Y_{t,t-1}o_{t-1}) \\
S_T^2(B_{1:t-1}) = \log p(Y_{t,t-1}\mid B_{1:t-1}o_{t-2}), \tag{13}
\]

and \( C_t \) is a constant to balance equation (12).

- \( S_T^1(\cdot) \): \( S_T(\cdot) \) is the temporal consistency score in equation (12) that is computed in a recursive manner.
- \( S_T^2(\cdot) \): As is depicted in equation (6), \( S_T^2(B_{1:t-1}) \) stands for the observation score in frame \( t-1 \). Based on equation (6), \( S_T^2(\cdot) \) can be similarly defined as \( S_T^2(B_{1:t-1}) = \sum_{j=0}^{M} f_A(b'_i) + \lambda_1 \sum_{j=1}^{M} g_A(b'_i, b'_j) \), where the previously tracked results \( b'_i, j \in \{1, ..., M\} \), are exploited to avoid a closed loop in the inference process.

- \( S_T^2(\cdot) \): \( S_T^3(B_t, B_{t-1}) \) is the transfer score for the tracked object to move from \( B_{t-1} \) to \( B_t \) when \( o_{t-1,t-1} \) is given. Here we let \( S_T^3(B_t, B_{t-1}) = \lambda_2 \sum_{j=0}^{M} g_T(b'_i, b'_j) \), which measures the overall transfer scores from the state \( B_{t-1} \) to the state \( B_t \), where \( g_T(\cdot) \) is a function (equation (14)) for calculating the transfer score of the \( j \)-th fragment. \( g_T(\cdot) \) is defined as

\[
g_T(b'_i, b'_j) = \begin{cases} 
    \langle c'_i, \phi_t(b'_i, b'_j, p'_i) \rangle & o'_i-t-1 = 0 \\
    0 & o'_i-t-1 = 1 \tag{14}
\end{cases}
\]

where we ignore this score if the corresponding fragment is detected occluded, \( c'_i \) is the linear SVM coefficient for the feature vector \( \phi_t(\cdot) \), and \( \phi_t(\cdot) \) is the temporal relation vector, i.e.,

\[
\phi_t(b'_i, b'_j, p'_i) = \begin{pmatrix} 
    dx' \times (dx')^2, dy', (dy')^2 \end{pmatrix}^T \\
dx' = x'_i - x'_{j-1} - p'_i[1] \\
dy' = y'_i - y'_{j-1} - p'_i[2] \tag{15}
\]

This term is exploited to measure the compatibility between the optical flow and the estimated object motion, in which \( p'_i \) stands for the estimated displacement vector between frame \( t \) and frame \( t-1 \) via the optical flow method \([35]\).

### C. Model Inference

After \( S_A(\cdot) \) and \( S_T(\cdot) \) are determined, we can obtain \( \hat{B}_t \) via equation (4). To be specific, we adopt the dynamic programming algorithm (Section 4.1.1 in [36]) to perform this optimization process. Paper [36] describes a dynamic programming algorithm defined on a tree, which aims to solve the following optimization problem

\[
L^* = \arg\min_L \left( \sum_{p=1}^{N} m_p(l_p) + \sum_{(p,q) \in s} d_{pq}(l_p, l_q) \right), \tag{16}
\]

where \( S_A(\cdot) \) and \( S_T^3(\cdot) \) correspond to identical probabilistic formats (equations (6) and (13)), so they have identical function formats.
where $m_p(\cdot)$ and $d_{pq}(l_p, l_q)$ denote the unary and binary terms, $L = (l_1, \ldots, l_N)$ denotes a configuration of the tree structured model. We refer the readers to Section 4.1.1 in [36] for more details.

We note that the recursive energy function is initialized based on ground truth in the first frame, and the occlusion labels of different fragments are initialized to be 0 (only $S_A(\cdot)$ needs to be initialized, which can be obtained by the SVM classifier trained in the first frame).

In Figure 2, we illustrate a toy model on how matching scores in previous frames are propagated to frame $t$ via equation (12). In this model, states of fragments in different frames are related with temporal relation vectors. As equation (12) is computed in a recursive way, scores obtained in previous frames can be directly exploited by the subsequent transfer process, which is computationally efficient.

Algorithm 1: Stage 1 (estimating the optimal bounding box $B_{1:1}$)

**Input:** occlusion state $o_{1:t-1}$ and $o'_{1:t-2}$ computed in the $(t-1)$-th and $(t-2)$-th frames respectively, the previously computed temporal consistency score $S_T(B_{1:1-1})|o_{1:t-2}$ under prior $o_{1:t-2}$, $S_T(B_{1:1-t})$, classifier coefficients $\{w_k\}_{k=1:T+1}$

1: if $|o_{1:t-2}-o'_{1:t-2}|2 = 0$
2: $S_T(B_{1:1}) = S_T(B_{1:1-1})o_{1:t-2} + S_T(B_{1:1-1}) + S_T(B_{1:1-1})$
3: compute the traceback table, i.e., given $B_{1:1}$, record the best position $B_{1:1-1}$
4: else
5: for $j \in \{1, 2, \ldots, T\}$
6: $S_T(B_{1:j}) = S_T(B_{1:j-1}) + S_T(B_{1:j-1})$
7: maintain the traceback table for the dynamic programming algorithm, i.e., given $B_{1:j}$, record the best position $B_{1:j-1}$
8: end
9: end
10: compute $S(B_{1:t})$ as in equation (4)
11: optimize $B_{1:T+1:t}$ = arg $\max_{B_{1:T+1:t}} S(B_{1:t})$ via dynamic programming algorithm

**Output:** $B_{1:T+1:t}$

\[ B_{1:T+1:t} = \arg \max_{B_{1:T+1:t}} S(B_{1:t}). \]

\[ S_B(o_{1:t}) = \log p(o_{1:t}|y_{1:t}, B_{1:t}) \]

\[ S_o(o_{1:t}) = \sum_{j=0}^{M} e_o(o_{1:j}), \]

\[ e_o(o_{1:j}) = \sum_{k=1}^{t-1} f_o(o_k') + \sum_{k=1}^{t-1} h_o(o_k', o_{k+1}) \]

\[ f_o(o') = 0 \text{ if } o = 0, \]
\[ f_o(o') = \frac{d_x}{d_y + d_z} \text{ otherwise} \]

**D. Scale Estimation**

The previous tracking process just tries to estimate the optimal target position, and does not take target scale into consideration. In this section, we further refine our tracking results by estimating the target scale. Specially, we first estimate the displacement (i.e., center position) of the target via the above method. Then, we refine the scale of it by sampling candidate scales on the basis of the estimated center and the previous scale. Here, we use an affine parameter $A = [x_t, y_t, s_t, \theta_t, \alpha_t, \phi_t]$ to denote the estimated bounding box, where $x_t, y_t, s_t, \theta_t, \alpha_t$ and $\phi_t$ denote the $x, y$ translation, scale, rotation angle, aspect ratio, and skew direction. We generate $N_s$ vectors $p_n = [0, 0, \delta_n, 0, 0, 0]_{N_s=1}^{N_s}$, and then states of $N_s$ candidate boxes are obtained as

\[ A_n = A + p_n, \quad n = 1, \ldots, N_s. \]

We obtain the best state from the candidates via

\[ \hat{A} = \arg \max_{A_n} \{u_t, \Phi(A_n)\}, \]

where $u_t$ denotes the SVM coefficients, $\Phi(A_n)$ denotes feature vector extracted in the bounding box specified by $A_n$. Once the scale for one frame is determined, we fix it for the following procedures.

We summarize the procedure of Stage 1 in Algorithm 1, where variable $T$ denotes the number of frames we trace back in the inference process.

**E. Occlusion Model**

In this work, the occlusion model is developed in a discriminative manner rather than a generative manner, i.e., model the posterior probability $p(o_{1:t}|y_{1:t}, B_{1:t})$ directly rather than model the likelihood and prior separately. To be specific, we introduce an occlusion score function,

\[ S_o(o_{1:t}) = \log p(o_{1:t}|y_{1:t}, B_{1:t}) \]

We obtain the best state from the candidates via

\[ \hat{A} = \arg \max_{A_n} \{u_t, \Phi(A_n)\}, \]

where $u_t$ denotes the SVM coefficients, $\Phi(A_n)$ denotes feature vector extracted in the bounding box specified by $A_n$. Once the scale for one frame is determined, we fix it for the following procedures.

We summarize the procedure of Stage 1 in Algorithm 1, where variable $T$ denotes the number of frames we trace back in the inference process.

**E. Occlusion Model**

In this work, the occlusion model is developed in a discriminative manner rather than a generative manner, i.e., model the posterior probability $p(o_{1:t}|y_{1:t}, B_{1:t})$ directly rather than model the likelihood and prior separately. To be specific, we introduce an occlusion score function,

\[ S_o(o_{1:t}) = \log p(o_{1:t}|y_{1:t}, B_{1:t}) \]

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We summarize the procedure of Stage 1 in Algorithm 1, where variable $T$ denotes the number of frames we trace back in the inference process.
Algorithm 2: Stage 2 (estimating the optimal occlusion labels $\bar{o}_{t:t+1}$)

**Input:** target position $B_{t-T+1:t}$, \{$(S_k^F)_{k=t-T+1}^{t}$, $(S_k^B)_{k=t-T+1}^{t}$\} for $k$ number of fragments $M$

1: for $i = t: T$
2: obtain the observation feature \{$y_k^i_{t-i+1}\}^{M}_{i=0}$ according to the bounding box $B_{t-i+1}$
3: compute $S_o(o_{t-i+1}) = \sum_{j=0}^{M} [f_o(o_{t-i+1}^j) + h_o(o_{t-i+1}^j, o_{t-i+1}^j)]$ + $S_o(o_{t-i+1})$
4: maintain the traceback table, i.e., given $o_{t-i+1}$, record the best occlusion state $o_{t-i}$
5: end
6: optimize $\bar{o}_{t-T+1:t} = \arg\max_{o_{t-T+1:t}} S_o(o_{1:t})$

**Output:** $\bar{o}_{t-T+1:t}$

where $d_1$ denotes the distance between the observation feature $y_k^i$ (corresponding to the bounding box $b_k^i$) and the foreground set $S_k^F$, and $d_2$ is the distance between $y_k^i$ and the background set $S_k^B$, i.e., $d_1 = \min_{y \in S_k^F} \|y_k^i - y\|_2$, $y \in S_k^F$ and $d_2 = \min_{y \in S_k^B} \|y_k^i - y\|_2$.

- $h_o(\cdot)$: $h_o(\cdot)$ depicts the temporal consistency of occlusion labels within two consecutive frames, which is defined as

\[
h_o(o_k^j, o_{k+1}^j) = \begin{cases} 1 & \left(1 + \|y_k^i - y_{k+1}^j\|_2\right) \leq 0 \\
0 & o_k^j = o_{k+1}^j \\
1 & o_k^j \neq o_{k+1}^j \end{cases}
\]

By combining equations (20) and (21), we can obtain the following recursive calculative formula,

\[
S_o(o_{1:t}) = \sum_{j=0}^{M} \left[ f_o(o_j^i) + h_o(o_{j-1}^i, o_j^i) \right] + S_o(o_{1:t-1}).
\]

Thus, we also adopt the dynamic programming algorithm to conduct the optimization problem (24) for estimating occlusion labels. Figure 3 shows the tracking results with and without occlusion labels, which illustrates the effectiveness of the proposed occlusion model. We show the procedure of Stage 2 in Algorithm 2.

**F. Model Update**

1) **Update SVM classifier:** This paper updates the concatenated SVM coefficients $w_{t+1} = \left[ u_{t+1}; v_{t+1}; c_{1_{t+1}}; \cdots; c_{M_{t+1}}; v_{t+1}; \cdots; v_{M_{t+1}} \right]$ every five frames. As discussed in [15], computing $w_{t+1}$ simultaneously may lead to the over-fitting problem. Thus, in this work we divide the training (and updating) process into two kinds of sub-problems (visualized in Figure 4). First, we concatenate the feature vector of fragment 0 with the relation vectors $\phi_s(\cdot)$ and $\phi_t(\cdot)$ and compute the classifier coefficients, i.e., $\left[ u_0^0; v_{t+1}; v_{t+1}; \cdots; v_{M_{t+1}}; c_{t+1}^0 \right]$. Second, we compute the classifier coefficients for the feature vector that is generated by concatenating the feature vector of fragment $j$ with its connected temporal relation vector $\phi_t(\cdot)$, i.e., $\left[ u_j^0; c_{j+1}^0 \right], j \in \{1, \ldots, M\}$. We extract samples whose occlusion labels are 0, and combine them with old support vectors to form the feature pool $\Psi_n$. Then the training (and updating) process is formulated as

\[
\arg\min_{\beta, \xi_n} \frac{1}{2} \beta^T \beta + C \sum_{n} \xi_n
\]

s.t. $\forall n \in \text{pos}$, $\beta^T \Psi_n \geq 1 - \xi_n$

$\forall n \in \text{neg}$, $\beta^T \Psi_n \leq -1 + \xi_n$

$\forall k \in K$,$ \beta[k] \leq 0$

(25)

where $\beta$ is the classifier coefficient of the concatenated feature, and $K$ is the set of coefficients corresponding to the feature element $(dx)^2$ and $(dy)^2$ defined in $\phi_s(\cdot)$ and $\phi_t(\cdot)$. We refer the reader to [37] for more details.

In each sub-problem, we directly exploit the tracked configurations in previous frames as positive samples. For generating the negative samples, we generate configurations by moving the position of the holistic fragment (or fragment $j$ in the second training process) from its tracked position and remain the spatial/temporal relationships among the fragments. Then the negative samples are extracted based on these newly generated configurations.

In Figure 5(a)(b)(c), we illustrate the learned coefficients for spatial and temporal relation vectors in a representative frame. Obviously, the reliable fragments (i.e., fragments 1 and 2), which contain fewer pose changes, contribute more to the holistic target as they are assigned more weights to. In addition, they exploit the temporal continuity more effectively for the reason that optical flow estimation is more accurate in them. In Figure 5(d)(e)(f), we show an example on how the weights of our motion model change when the abrupt motion occurs. Altogether three frames are presented, and

Fig. 3. Tracking results with (b) and without (a) occlusion model. In each group, the confidence map is shown on the left, and the image frame with the tracked box is shown on the right.

Fig. 4. A demonstration of the concatenated features for two kinds of training, in both of which we concatenate feature vectors located in the dashed circle. The red and blue lines represent spatial and temporal relation vectors respectively.
large-scale motion occurs near frame 106 and 111. Generally speaking, the weights of our motion model become relatively smaller when abrupt motion occurs. The motion model is less reliable in such cases (caused by the inaccurate optical flow estimation), and thus is assigned to smaller weights. This figure validates that our model is able to capture scene changes, and learns proper coefficients for each term.

2) Update the spatial relation vector: After obtaining the optimal state \( \hat{B}_t \) in the current frame, the spatial relation vector in equation (9) can be updated as

\[
d_{t+1} = (1 - \alpha) d_t + \alpha \left[ x_t - \hat{x}_t; y_t - \hat{y}_t \right],
\]

where \( \alpha \) is the update rate to integrate the old spatial relation vector and the newly tracked spatial configuration. \( (x_t, y_t), j \in \{1, ..., M\} \), stands for the optimal \( x \), \( y \) positions for different fragments in the \( t \)-th frame.

IV. EXPERIMENTAL EVALUATION

In this work, the proposed tracking algorithm is implemented in MATLAB on a PC machine with Intel i7-3370 CPU (3.4 GHz) and 32 GB memory. The tracked object is divided into three fragments vertically or horizontally according to aspect ratio of the target. Specially, according to the annotation in the first frame of each sequence, we divide the target into three fragments horizontally when the width is 1.6 times larger than the height, and vertically otherwise. Basically, we adopt the HOG (Histogram of Gradient) features to describe each divided fragment and the holistic template, which are normalized to unit vectors. For the tracker based on HOG features, we perform the target search within a radius of 30 pixels, which is the same as the radius in the negative samples extraction process. The parameters \( \lambda_1, \lambda_2 \) and \( \alpha \) are respectively set to 0.005, 0.001 and 0.2. The number of frames we trace back in the inference process for both bounding box and occlusion label is set to 5. In addition to the HOG features, we also implement a version of our tracker by exploiting the color features from [38]. The target search region is enlarged in this version for robustness. We exploit the online setting for the proposed trackers. The source codes of this paper will be made publicly available.

A. Quantitative Evaluation

We evaluate the performance of our method on the recent published benchmark [32] against 36 state-of-the-art trackers (listed in Table II), including VTD [39], Stuck [40], MIL [41], SCM [42], TLD [35], CSK [43], PBT [15], TGPR [44], KCF [45], DSST [46], Diag [47], MEEM [38], FCNT [48] and so on. One-pass evaluation (OPE) is exploited for all trackers, and the results are presented in both precision and success plots. The precision plot is exploited to show the percentage of frames in which the distance between the estimated target location and the ground-truth is within a varying threshold. Similar with the precision plot, the success plot shows the percentage of successfully tracked frames with different thresholds. In precision plot, the results at error threshold of 20 are exploited for ranking, while in the success plot, the AUC (area under curve) scores are used to rank the performance.

**Table II**

<table>
<thead>
<tr>
<th>TRACKING ALGORITHMS TO BE COMPARED WITH OURS. NOTE THAT THE VIVID TESTBED [49] CONTAINS 4 TRACKERS.</th>
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<td>VTS [53]</td>
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**Overall Performance.** The overall performance (evaluated on 51 sequences in the benchmark) of different trackers is
demonstrated in Figure 6. In this section, we report the performance of two versions of our trackers “Ours_hog” and “Ours_color”. For “Ours_hog”, we exploit the HOG features, and for “Ours_color”, we further enhance the tracker by exploiting the color features (from [38]) in the fragments. Overall, the proposed trackers perform favorably against all other compared trackers in terms of OPE criterion. The tracker “Ours_color” achieves an 88.4% distance precision rate, which improves the second best tracker FCNT by 2.8%. Our method also improves the FCNT tracker by 1.5% in success plots. In addition, among the trackers that take the gray scale images as input, our tracker “Ours_hog” performs 3.6% and 3.1% better than the second best ones in precision and success plots respectively.

Attribute-based Evaluation. It is also very valuable to evaluate different trackers on different subsets (corresponding to different challenging factors) of the benchmark. In [32], the challenges of all 51 image sequences are divided into 11 attributes, which cover the common challenging factors in visual tracking (such as occlusion, deformation, background clutters and so on). We conduct the attribute-based evaluation by comparing different trackers on 11 attributes and report eight of them in both precision and success plots (Figure 7 and Figure 8). It is obvious that our trackers achieve very good performance in all reported attributes. Specially, in the “occlusion” attribute in success plots, the proposed tracker “Ours_color” performs 2.9% better than the FCNT method and 3.7% better than the MEEM method. In the “out-of-plane rotation” and “in-plane rotation” attributes in success plots, our tracker achieves 1.7% and 3.2% better performance than the second best ones respectively. In addition, among the trackers that take the gray scale images as input, our tracker “Ours_hog” also has the best performance in the “occlusion”, “out-of-plane rotation”, “deformation”, “fast motion” and “motion blur” attributes in success plots. The underlying reason for the performance improvement is that the proposed method exploits both spatial and temporal continuity (and
Fig. 9. Performance comparison for different components of the proposed method.

discontinuity) effectively.

B. Evaluation on Different Components

We evaluate the effectiveness of different components to emphasize their contributions. For comparisons, we generate five variants of the proposed tracker: (1) the “baseline” method: the method that only exploits spatial information (i.e., fragment-based appearance model) without considering the occlusion labels and the temporal consistency score; (2) the “baseline+o” method: the “baseline” method with the occlusion model; (3) the “baseline+t” method: the “baseline” method with the temporal consistency model; (4) the “baseline+o+t” method: the method that simultaneously considers temporal continuity and discontinuity (i.e., our method without scale estimation); (5) the “Ours_thres” method: the method that exploits the occlusion model and the thresholded motion model. For convenience, the following experiments are based on the HOG features. The performance of our method and its variants is illustrated in Figure 9.

1) Evaluation on Scale Estimation: We evaluate the proposed scale estimation process by comparing “Ours_scale” and “baseline+o+t”. As is shown in Figure 9(a)(b), the scale estimation process, though very simple, works well in improving the tracking precision and robustness. Specially, the method with scale estimation process improves the “baseline+o+t” method by 1% and 2.5% in the overall precision and success plots. The improvement is more remarkable if we make comparisons between these two methods on the sequences that are annotated to have large scale variations (Figure 9(c)(d)), where the improvements in precision and success plots are respectively 6.2% and 9.3%. This experiment validates the effectiveness of the scale estimation process.

2) Evaluation on Occlusion Model: As is described in previous sections, the occlusion model plays a very important rule in the visual tracking system. In this subsection, we test its effectiveness by comparing two variants of our algorithm, i.e., “baseline+o+t” and “baseline+t”. We perform experiments on both the entire benchmark video sequences (Figure 9(a)(b)) and the sequences annotated to have the occlusion attribute (Figure 9(e)(f)). By exploiting the occlusion model, we improve the overall performance by 2.4% and 3.0% in precision and success plots. What is more, in the video sequences with the occlusion attribute, the improvement is expanded to 6.8% and 5.8%.

3) Evaluation on Temporal Consistency Model: In this subsection, we test the effectiveness of our temporal consistency model by comparing the two methods “baseline+o+t” and “baseline+o”, whose performance is visualized in Figure 9(a)(b). From this experiment, we see that the temporal consistency information improves the overall performance by 2.8% and 4.1% in precision and success plots respectively. In addition, by making comparisons between “baseline+o+t” and “Ours_thres”, we see that the proposed temporal consistency model has superior performance over the thresholded motion model.

4) Analysis: From the above experiments, both “baseline+o” and “baseline+t” methods improve the “baseline” method, however, these improvements are limited. The underlying reasons are two folds: (1) most sequences obey the temporal continuity assumption, thus, the “baseline+t” method can improve the performance. But temporal continuity assumption cannot be ensured in many cases, especially when the target is occluded. In these cases, the “baseline+t” method drops the tracking performance. (2) Although the “baseline+o” models the occlusion explicitly and improves the tracking performance when the tracked object suffers from occlusions, it may break the temporal continuity, and pull down the performance in some sequences where the role of the temporal continuity is dominant. In addition, the miscalculated
occlusion state can not be revised in the tracking process. The “baseline+o+t” method provides a unified framework to combine the fragment-based observation, occlusion, temporal continuity models, thus, achieving a good performance.

C. Evaluation of Different Fragment Numbers

In this subsection, we conduct an experiment to test the influence of the number of fragments in our appearance model. In this experiment, we vary the number of fragments from 2 to 5, and name the corresponding trackers as “Ours_2”, “Ours_3”, “Ours_4”, and “Ours_5”. We evenly divide the holistic target into 2 (or 3) non-overlapping fragments, while for “Ours_4” and “Ours_5”, we evenly divide the holistic target into 4 (or 5) overlapping fragments with each fragment size being 1/3 of the holistic target. We do this to avoid that a fragment contains too small area in “Ours_4” and “Ours_5”. In addition, we also slightly adjust the parameter $\lambda_1$ in this experiment to re-balance the holistic target and fragments. The experimental results for our algorithm with different numbers of fragments are visualized in Figure 10(a)(b). We can see that the number of fragments does influence the tracking performance to some extent. However, when the number of fragments is larger than 3, the influence is limited.

D. Comparison with the Gaussian Random Walk Process

In this subsection, we show the impact of the proposed recursive model when compared to using just a Gaussian random walk process. A new version of our method “Ours_gaussian” is implemented, which exploits the particle filter approach as the motion model. For “Ours_gaussian”, we firstly estimate the target position in an iterative manner. With the computed occlusion prior, a recursive inference is performed in the spatial-temporal structure, by which the positions of the tracked object (and fragments) of several frames are simultaneously optimized. For conducting occlusion estimation, a discriminative occlusion model is proposed, which directly compares the target with the positive and negative samples with an $L_2$-norm distance. The temporal consistency of occlusion states among frames is also taken into consideration for optimizing. In addition, a simple yet effective training (and updating) strategy is also introduced to ensure the model coefficients are properly learned. Finally, we conduct extensive experiments on the recent benchmark dataset, on which the results demonstrate the superiority of the proposed tracker over other competing trackers.

V. Conclusion

This paper presents a novel tracking algorithm by estimating the position of the tracked object and occlusion state in an iterative manner. With the computed occlusion prior, a recursive inference is performed in the spatial-temporal structure, by which the positions of the tracked object (and fragments) of several frames are simultaneously optimized. For conducting occlusion estimation, a discriminative occlusion model is proposed, which directly compares the target with the positive and negative samples with an $L_2$-norm distance. The temporal consistency of occlusion states among frames is also taken into consideration for optimizing. In addition, a simple yet effective training (and updating) strategy is also introduced to ensure the model coefficients are properly learned. Finally, we conduct extensive experiments on the recent benchmark dataset, on which the results demonstrate the superiority of the proposed tracker over other competing trackers.
assumed Gaussian distribution, which is usually exploited in modeling motions between two consecutive frames. With these estimated model parameters, we compute the ideal cumulative distribution function for the Gaussian distribution (shown in the red line in Figure 12). Then, we compute the empirical cumulative distribution function for the horizontal motions of the benchmark video sequences (shown in the blue line in Figure 12). At last, we perform the Kolmogorov-Smirnov test on these two cumulative distribution functions and determine if these two distributions are significantly different. The computed cutoff value and test statistic under significance level 5% are respectively 0.0095 and 0.1470. As 0.1470 > 0.0095, the Kolmogorov-Smirnov test rejects the hypothesis that the motions obey the Gaussian distribution. In addition, we also exploit the Lilliefors test (a version of the KS test) for analysis. The computed cutoff value and test statistic are 0.0053 and 0.1470, so the Lilliefors test also rejects the hypothesis that the motions obey the Gaussian distribution. In Figure 12, we show the estimated Gaussian distribution cannot fit the data very well.

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