ML-FOREST: A Multi-label Tree Ensemble Method for Multi-Label Classification

Qingyao Wu, Mingkui Tan, Hengjie Song, Jian Chen, Michael K. Ng

Abstract—Multi-label classification studies the problem where each example is associated with multiple class labels. Since the labels are often dependent to other labels, exploiting label dependencies can greatly help to improve the multi-label classification performance. The label dependency in existing studies is often given as prior knowledge or learnt from the labels only. However, in many real applications, such prior knowledge may not be available, or the labeled information might be very limited. In this paper, we propose a new algorithm, called ML-FOREST, to learn an ensemble of hierarchical multi-label classifier trees to reveal the intrinsic label dependencies. In ML-FOREST, we construct a set of hierarchical trees, and develop a label transfer mechanism to identify the multiple relevant labels in a hierarchy way. In general, the relevant labels at higher levels of the trees capture more discriminable label concepts, and they will be transferred into lower level children nodes, which characterize label concepts that are harder to discriminate. The relevant labels in the hierarchy are then aggregated to compute the label dependency and make classification prediction. Our empirical study shows encouraging results of the proposed algorithm in comparison with the state-of-the-art multi-label classification algorithms under Friedman test and post-hoc Nemenyi test.

Index Terms—Multi-label classification, label dependency, label transfer, tree classifier, ensemble methods.

1 INTRODUCTION

MULTI-LABEL classification aims to predict the presence or absence of certain labels of an example which is associated with multiple classes. Different from classical multi-class problems, where an example is associated with only one single label, the multi-label classification is more general since real-world objects often contain multiple semantic objects. For example, a real-world image usually belongs to multiple categories based on different context, such as water, ship, etc.; while a text document can be classified into a set of topics, such as *news*, *sports*, etc. In the last decades, multi-label classification problem has received broad attention from various research domains, such as text categorization [1], [2], [3], bioinformatics [4], [5], [6], and computer vision [7], [8], [9].

A straightforward multi-label classification approach is the *binary relevance* (BR) [10], which decomposes the problem into a set of single-label multi-class problems. In this way, existing multi-class classifiers are leant and then applied to do prediction. This simple method, however, totally neglects the dependencies among multiple labels. In practice, multiple objects in an example (such as an image) may have strong relations or dependencies. For example, if *ship* category is presented in an image, it is very likely that the *water* category is also in that image. Exploiting such label dependency may significantly improve the prediction performance for multi-label classification.

Plenty of previous studies [11], [12] have tried to exploit the label dependency to improve the prediction performance. However, how to effectively model the label dependency explicitly is still a challenging problem. In [13], the authors simply assume that the label dependency is provided as prior knowledge by external resources e.g., a hierarchical label structure. However, such prior knowledge is often unavailable in real-world situations. Some other approaches, like [14], [15], consider learning the label dependency from very limited information, e.g., the co-occurrence of the labels in the training set. However, these kind of learning methods may cause over-fitting issues [11].

In this paper, we propose a new tree ensemble algorithm, called *Ml-Forest*, to explicitly exploit the label dependency for multi-label classification. In ML-FOREST, a set of hierarchical trees are constructed to learn the label dependency, and then combined as an ensemble to do multi-label prediction. In other words, the primary focus of this paper is to find a good hierarchical structure so that two relevant instances of with strong label dependency will be located in the same node of the tree. To achieve this, we design a new tree generation algorithm to partition the learning data into smaller subsets from the root to the leaves, and then identify relevant labels for each node with a label transfer mechanism.

For the first task of the algorithm, we train multi-class classifiers at each node to divide the data into child nodes. Here, each data instance is partitioned into one child node according to the classifier prediction results, and the class label with highest probability given at the node is considered as its relevant label.

For the second task of the algorithm, a label transfer mechanism is involved to recursively propagate the relevant labels from the root down to the leaf node. For example, if a relevant label is found at a node, all of its child nodes would automatically belong to this relevant label, and we seek a new relevant label (if any) which respects the label dependencies of the instances in the child node. In the end,

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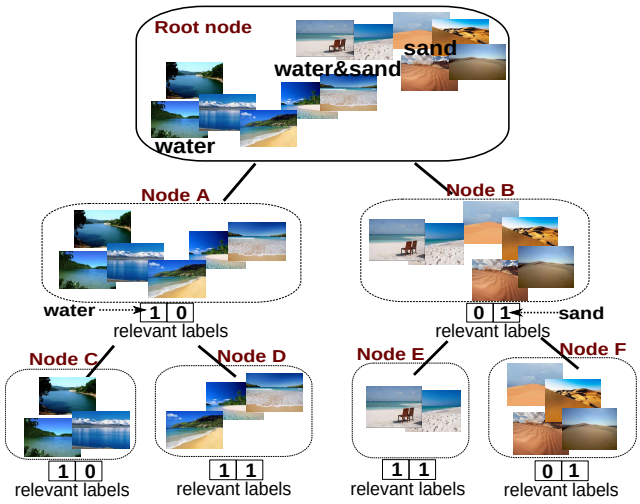

Fig. 1. An example of a hierarchical tree for scene classification.

each leaf node is characterized by multiple relevant labels given by the nodes at different levels of the tree. This leads to a new label dependency representation, where the learning models at different levels work together effectively to reveal multiple label concepts belonging to the given data. Intuitively, the relevant labels at high levels in the hierarchy may tend to capture "more significant" label concepts and hence are thematically more general, while the relevant labels at low levels would capture "less significant" label concepts and hence are thematically more specific.

In Fig. 1 we illustrate the above ideas by showing an example of a hierarchical tree constructed in a multi-label scene classification task. In this figure, the instances at the root node belong to *water* or *sand* or both these two classes simultaneously (e.g., a beach scene contains both water and sand). A hierarchical tree then is constructed to partition the data from the root to the leaf nodes, and we identify the relevant labels at each node to capture the label concepts based on the label transfer mechanism. In particular, the data instances are partitioned into the same node if they are close to each other and they would be labeled hierarchically such that the label concepts at the higher layers are often more easier to characterized.

In the above example, part of the beach images would be partitioned into node **A** and labeled as *water* at the first layer of the tree; while the rest beach images would be labeled as *sand* in node **B**, depending on which class the images belong to. The resulting class decision hyperplanes will further split the data of node **A** into nodes **C** and **D**. It is worth mentioning that the *water* class associated to node **D** is inherited from node **A**, whilst the sand class associated to node E is given by node **B**. For convenience, hereafter this hierarchical multi-label tree is referred to (*Ml-Tree*).

The major contributions of this paper are as follows.

- We propose a new hierarchical tree algorithm, called ML-TREE, to solve the multi-label classification task. Unlike the BR method which transforms the data into independent binary problems, our algorithm exploits the intrinsic label dependency of the data and incorporates the ML-TREE structure to find the relevant labels of an instance with multiple labels. Therefore, the proposed approach provides a principled way for modeling the intrinsic label dependency of the data into a tree structure.
- We design a label transfer mechanism to find the relevant labels in the hierarchy. The labels of the high levels in the hierarchy will be used as priors for the nodes in the low levels to reduce the label space. Therefore, building the classifier model for low levels can be very efficient.
- We develop an ensemble strategy to construct multiple hierarchical multi-label trees and combine the predictions of different trees as an ensemble to make predictions.
- We evaluate the empirical performance by conducting an extensive set of experiments on real-world problems in text classification, computer vision and bioinformatics. Experimental results have demonstrated that the ML-FOREST approach is highly competitive to the state-of-the-art approaches under Friedman and Nemenyi tests [16].

The rest of this paper is organized as follows. The problem of multi-label classification and related work are introduced in Section 2. The proposed methodology is then described in Section 3. The data sets, the experimental setup and experimental results are discussed in Section 4. Finally, conclusions are drawn in Section 5.

2 Related Work

Let $\mathcal{X} = \mathcal{R}^d$ be the d-dimensional input space. Given a labeled data set $\{(\mathbf{x}_i, \mathbf{y}_i)\}_{i=1}^m$, where $\mathbf{x}_i \in \mathcal{X}$ contains d input features and $\mathbf{y}_i \in \mathcal{Y} = \{0,1\}^q$ consists of q possible labels, the multi-label learning aims to learn a hypothesis $f : \mathcal{X} \to \mathcal{Y}$ that maps an input $\mathbf{x} \in \mathcal{X}$ to outputs $\mathbf{y} \in \mathcal{Y}$. Regarding the label $\mathbf{y}_i$ of the i-th example, $y_i^j = 1$ if x_i contains the j-th target, and $y_i^j = 0$ otherwise.

In the past decades, a number of multi-label classification approaches have been developed regarding various areas, such as text categorization [1], [2], [3], bioinformatics [4], [5], [6], and computer vision [7], [8], [9]. These works have revealed that exploiting the dependency among different labels is crucial to improve the good performance for multi-label classification. For example, Zhang et al. [14] summarized the existing multi-label approaches into three categories based on the orders of dependencies exploited in the system, including *First-order* approaches, *Second-order* approaches and *Higher-order* approaches.

First-order approaches decompose the multi-label classification task into a number of independent tasks [17], [18]. The most common method is the *binary relevance* (BR) method [10], which transforms a multi-label problem into multiple separate and independent binary problems, one for each label. It is clear that the first-order methods are incapable of label dependency, which might cause a degradation of the predictive performance.

Second-order approaches consider the pairwise relations between labels, such as the interaction between any pair of labels [19], [20]. In general, such pairwise label dependency is estimated by the co-occurrence or some other equivalent measures of the labels. However, these approaches might

over-fit the training data since these dependencies are usually inaccurate.

High-order approaches consider even higher order of relations among labels, such as the *full-order* style imposed on all the labels [21], [22], [23], [24]. For example, Clare and King [25] applied the dependencies between all labels to enhance the multi-label classification performance. However, the *full-order* approaches are usually impossible when the number of labels is large, where the number of possible label subset combinations can be exponentially huge.

Hierarchical tree based model is a family of learning algorithms with simple theoretical foundation, and it has been widely applied in multi-label classification [22], [25], [26], [27], [28], [29], [30]. For instance, Clare and King [25] adapted the C4.5 algorithm for multi-label data by modifying the formula of entropy calculation. Comité et al. [31] learned a multi-label alternating decision trees from texts and data. Blockeel et al. [32] proposed predictive clustering trees (PCT) to make multi-target prediction/multi-label classification; while Vens et al. [23] introduced a high-order approach to extend PCT algorithm to deal with multi-label classification problem where these classes are organized in a hierarchy form. But the hierarchical label dependencies should be provided by external information as the prior knowledge. Tsoumakas et al. [22] proposed a HOMER algorithm using a tree structure to handle problems with a large number of labels, in which the whole label set is disjointed into subsets to construct the tree by using a balance clustering algorithm. This method is a high-order approach and it does not require the label structure as a prior knowledge, but it is computationally inefficent to fine-tuning the parameters involved in bulidng the hierarchical model.

Recently, various researchers (see [33], [34], [35])have exploited the random forest type ensemble methods for multi-label classification to enhance the learning performance. Motivated by recent progress in ensemble learning, we propose to exploit the label dependencies to improve the multi-label prediction performance via the ensemble of hierarchical trees, namely ML-FOREST. Our proposed ML-FOREST method is a *high-order* method, where each classifier tree addresses dependency among a subset of labels based on relevant labels generated from the root to the leaves. Note that in this way, the size of the label subset is much reduced when considering their dependencies, and such intrinsic label dependencies will be explicitly presented in the hierarchical trees. More importantly, the learnt dependency which offers a natural way to gain more insights into the multi-label classification, will lead to improvement in predictive performance and lower computational cost compared to other state-of-the-art tree based multi-label learning algorithms.

The proposed ML-FOREST algorithm is different from the PCT [32], HOMER [22], and TSA [36].

- In PCT [32], a variance function is employed to split the learning data by maximizing the cluster homogeneity, and a prototype function is used to compute a label for each leaf. In our proposed method, classifier models are constructed to partition the data into child nodes, and we identify the relevant labels at each node and transfer the labels from the root to leaves in a top-down manner to preserve the label dependence in the hierarchy.
- Tsoumakas et al. [22] construct a hierarchy of multi-label classifiers (i.e., HOMER) to handle the task with a large number of labels. HOMER starts with a root node containing all the possible classes, and follows a recursive process to partition the classes into the leaves (each class corresponds to one leaf). Each internal node contains the union of the label of its children. While our proposed algorithm recursively partitions learning data into child nodes, in which every internal node consists of all instances of its children. In addition, for a multi-label instance $\mathbf{x}$, HOMER forwards $\mathbf{x}$ into multiple leaves, and the union of the single-labels in the corresponding leaves is used as the multi-label output of the HOMER approach. On the other hand, in our proposed algorithm, the multi-label instance $\mathbf{x}$ is forwarded into only one leaf node, and the corresponding labels of nodes in the path from the root to this leaf are taken as the multi-label output of the proposed approach.
- Madjarov et al. [36] propose a Two Stage Architecture (TSA) algorithm for multi-label learning. The algorithm is implemented using two layers. In particular, binary relevance models are built in the first layer to reduce the complexity of the training of pair-wise models in the second layer. This method is a second-order approach. While in our proposed algorithm, we construct a hierarchical tree structure, which models the label dependency following the divide-and-conquer paradigm. We use a recursive process to partition the data into smaller subsets, and this process continues until the remaining instances at the node cannot be further split by the induced classifier. Therefore, our algorithm is a high-order approach that constructs multiple-layer models.

3 METHODOLOGY

In this section, we present the ML-FOREST method for multi-label classification in details. We first describe the classifier tree construction algorithm as well as the label transfer mechanism for exploiting label dependencies. Next, we incorporate the classifier trees into a forest via a new ensemble framework to further improve the prediction performance, and give the computational complexity of the proposed algorithm.

3.1 THE ML-TREE ALGORITHM

Statistically, the label dependency can be categorized into two groups, namely conditional and unconditional dependency. Here the conditional label dependency captures the dependency of the labels given a specific instance $\mathbf{x} \in \mathcal{X}$; while the unconditional label dependency is the expected dependency averaged over the marginal distribution of all instances [12].

The joint conditional probability distribution $p(\mathbf{y}|\mathbf{x})$, which specifies the probability of the label combination for a specific instance, provides a convenient point of departure

for analyzing the conditional label dependence. Mathematically, $p(\mathbf{y}|\mathbf{x})$ can be written as:

$$\begin{aligned} p(\mathbf{y}|\mathbf{x}) &= p(y^1|\mathbf{x})p(y^2,\cdots,y^q|y^1,\mathbf{x}) \\ &= p(y^1|\mathbf{x})p(y^2|y^1,\mathbf{x})p(y^3,\cdots,y^q|y^1,y^2,\mathbf{x}) \\ &= p(y^1|\mathbf{x})p(y^2|y^1,\mathbf{x})\cdots p(y^q|y^1,\cdots,y^{q-1},\mathbf{x}) \end{aligned} \quad (1)$$

where the elements of $\mathbf{y} = (y^1, y^2, \ldots, y^q)$ can be arranged by arbitrary order. Based on the above formulation, the joint conditional probability $p(\mathbf{y}|\mathbf{x})$ can be estimated by multiple steps, with each step for one class. To be more specific, we can build a model to compute a prediction probability for one label given $\mathbf{x}$ (e.g. $p(y^1|\mathbf{x})$), then the output is used as the prior to help the estimation of the probability for another label (e.g. $p(y^2|y^1,\mathbf{x})$).

Recently, researchers have considered the classifier chain idea (see [37], [38]) to model the underlying label dependency, such as Probabilistic classifier chain (PCC) [37] which is a representative algorithm to estimate the conditional joint distribution, and the classifier chain (CC) [38] algorithm which can be regarded as a deterministic approximation of probability only using $\{0, 1\}$ values [37]. All these two algorithms build q classifiers for estimation w.r.t. q class labels, in which the j-th classifier is used to estimate $p(y^j = 1|\mathbf{x}, y^1, \cdots, y^{j-1})$ and the result is further propagated to the $(j+1)$-th classifier by constructing a new feature vector augmented by the value of j-th label. The $(j+1)$-th classifier is to estimate $p(y^{j+1} = 1|\mathbf{x}, y^1, \cdots, y^j)$ and it generates a new feature vector $(\mathbf{x}, y^1, \cdots, y^{j+1})$ for the next classifier. It has been shown that the order of labels to be computed in PCC and CC has a great effect on the classification performance, and how to determine the order of labels is still an open question [38].

Motivated by recent progress in exploiting label dependency, in this paper, we propose a new hierarchical tree algorithm, called ML-TREE, which explicitly considers the intrinsic label dependency in a hierarchy way. The pseudo-code of the algorithm is described in Algorithm 1 and 2. More specifically, there are three folds constructing the hierarchical structure: 1) At each internal node, a multi-class classifier model is built to partition the training data into smaller subsets according to the predictions of the model; 2) A set of relevant labels are identified at each node for multi-label classification; 3) The relevant labels of a node in high levels will be transferred into its child nodes, which consider the remained labels according to the classifier built in this phase.

Our goal is to find the relevant labels to be associated with the data examples at each node. If no examples are found, the majority class of its parent node is returned; otherwise, the majority class of all examples at the node is returned. After that, the relevant labels identified by the nodes at higher levels are transferred into the nodes at lower levels as prior label information. This major step of ML-TREE, i.e., SPLITTEST, is detailed as follows.

i) When the ML-TREE function begins, it actually invokes itself recursively for each partition using the SPLITTEST function to train a multi-class classifier and identify the relevant labels. At each node splitting, SPLITTEST builds a group of one-against-all binary classifiers [39] for those

Algorithm 1 ML-TREE

Input: A training data set $\mathcal{D}$, and a relevant label vector $\mathbf{b}$ = none
Output: A hierarchical multi-label tree

```
1: (b, h, P) = SplitTest(D, b)
2: if h ≠ none ∧ Acceptable(P) then
3:    for D_i ∈ P do
4:       tree_i=ML-Tree(D_i, b)
5:    end for
6:    return node(h, b, ∪_i{tree_i})
7: else
8:    return leaf(h, b)
9: end if
```

Algorithm 2 SPLITTEST

Input: A training data set $\mathcal{D}$, a relevant label vector $\mathbf{b}_p$ from parent
Output: A classifier h, a new relevant label vector $\mathbf{b}$, and a partition $\mathcal{P}$ for current node

```
1: compute p using Eq. (2)
2: compute b using Eq. (3) and (4)
3: (h, P) = (none, φ)
4: h = build classifier on D for those labels which have not
   been identified according to b
5: if h ≠none then
6:    P= partition D using h
7: end if
8: return (b, h, P)
```

remained labels that have not been identified in any of its parent nodes, i.e., the labels with $b^j = 0$ (see Line 4 in Algorithm 2) . Then, each example is classified into one class with maximum confidence score from the multi-class classifier, and it is partitioned into the corresponding child node in below layer (see Line 6 in Algorithm 2). Note that it is possible that the confidence scores of two (or more) classifiers might be equally maximal. In this case, the example is classified into the class with largest prior.

ii) In order to find the relevant labels for each node, we design a a label purity vector, denoted by $\mathbf{p} = [p^1, \cdots, p^q]^\top$, to represent the purities of different classes. Specifically, we calculate each class label's data purity by

$$p^j = \frac{1}{|\mathcal{D}|}\sum_{\mathbf{x}_i \in \mathcal{D}} y_i^j, \quad (2)$$

where $p^j \in [0, 1]$ is the purity for the j-th class label, $\mathcal{D}$ is the examples at the node, and $|\mathcal{D}|$ is the number of examples in $\mathcal{D}$.

Then we construct a relevant label vector, $\mathbf{b} = [b^1, \cdots, b^q]^\top$, and incorporate the purities into its calculation to seek the majority labels as the relevant labels of a node.

$$b^j = \begin{cases} 1, & \text{if } p^j \geq \lambda, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

where b^j is the relevant label indicator for the j-th class label, $\lambda \in (0.5, 1.0)$ is a purity threshold.

iii) We also use a label transfer mechanism to transfer the result of the relevant label vector to below layer.

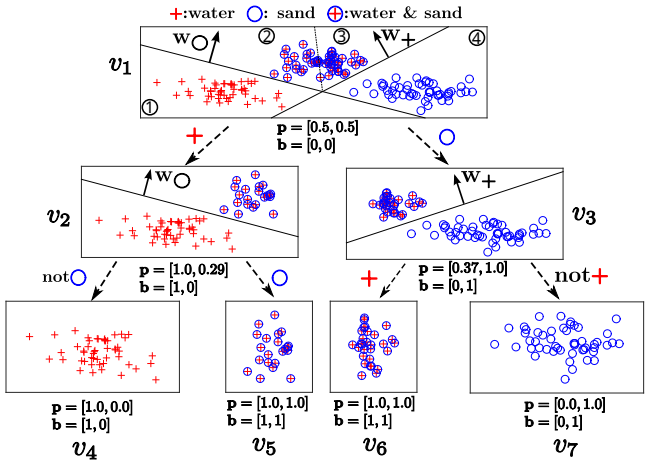

Fig. 2. An example of training procedure for multi-label classification.

Our idea is to preserve the identified relevant label vector $\mathbf{b}_p = [b_p^1, \cdots, b_p^q]^\top$ from the parent node and incorporate it as an additional indicator with the relevant label vector $\mathbf{b}_c = [b_c^1, \cdots, b_c^q]^\top$ of a child node, which can be obtained by (3), to obtain a final result of relevant labels $\mathbf{b}$ as follows:

$$b^j = \begin{cases} 1, & \text{if } b_p^j = 1 \text{ or } b_c^j = 1, \\ 0, & \text{otherwise.} \end{cases} \tag{4}$$

iv) The above process continues until a stopping criterion is reached, i.e., the data cannot be further split by the induced classifiers.

Fig. 2 shows an example of two-label problem regarding Fig. 1 to illustrate the construction of ML-TREE. Note that each node in ML-TREE is with a set of one-against-all classifiers which are used to partition the data into child nodes. Without lose of generality, we use linear SVM as the base classifier, and set the thresholding value $\lambda = 0.9$ as default value. As shown in Fig. 2, the root node v_1 contains all training instances, and we train two one-against-all classifiers (i.e., $\mathbf{w}_0$ and $\mathbf{w}_+$) for the "0" and "+" classes, respectively (see Line 4 in Alg. 2). We then classify the instance $\mathbf{x}$ according to the confidence scores, i.e., the margin values $(\mathbf{w}_0 \cdot \mathbf{x})$ and $(\mathbf{w}_+ \cdot \mathbf{x})$ (see Line 6 in Alg. 2). As illustrated in node v_1, the solid lines are the resulting decision boundaries given by the binary classifiers, and the dash line is the combined boundary decided by the relative magnitudes of margins. Specifically, when $(\mathbf{w}_0 \cdot \mathbf{x}) > (\mathbf{w}_+ \cdot \mathbf{x}) > 0$, which is the case in region ③, $\mathbf{x}$ will be classified as "0" class. While if $(\mathbf{w}_+ \cdot \mathbf{x}) > (\mathbf{w}_0 \cdot \mathbf{x}) > 0$, which is the case in region ②, $\mathbf{x}$ will be classified as "+" class. According to the decision surface, the instances in regions ① and ② would be classified as "+" class; while the instances in regions ③ and ④ would be classified as "0" class. Next, we use the prediction function to compute a vector of label probabilities $\mathbf{p}$ and a vector of relevant labels $\mathbf{b}$ w.r.t each child node (see Lines 1 and 2 in Alg. 2). For v_2, we have $\mathbf{p} = [1.0, 0.29]^\top$ and $\mathbf{b} = [1, 0]^\top$. According to the prediction criterion, the "+" class is considered as the relevant label for the instances in v_2. When building the classifier models for v_2, we do not need to consider the "+" class any more. In other words, we just need to train a classifier w.r.t. the "0" class for further splitting. The identified relevant label "+" will be transferred into the following child nodes, i.e. nodes v_4 and v_5. The construction process is similar for nodes v_3, v_6 and v_7 in the right sub-tree.

3.2 THE ML-FOREST ALGORITHM

To improve the prediction performance, we further propose a ML-FOREST algorithm which extends the tree model using an ensemble method. A single standalone tree model can be assumed to partition the whole data space into regions belonging to different classes. However, it is likely that one single tree may overfit the data in the local region. In particular, it may make the inference of a test example unreliable. By applying an ensemble of trees, we first partition the whole data set into multiple random data subsets, and then construct multiple trees for each subset. In this way, we can greatly reduce the risk of overfitting on training data, and thus generally increase the overall prediction performance. More importantly, such strategy can greatly improve the scalability of the method over large-scale data sets. Lastly, its complexity is linear w.r.t. the number of trees. The ML-FOREST algorithm which builds an ensemble of K classifier trees $\{T_1, \cdots, T_K\}$. for multi-label classification is described in Alg. 3,

It has been shown that an ensemble learner with excellent generalization accuracy has high diversity in component learners. The theoretical and practical studies of ensemble diversity are well documented [40], [41]. In order to achieve diversity, we employ two randomization procedures to generate multiple hierarchical trees in ML-FOREST. First, each tree is trained on a data subset randomly drawn from the entire training set $\mathcal{D}$ using *sampled with replacement* [42]. In addition, the purity thresholding value λ for each tree is selected randomly in the range $(0.5, 1.0)$. Such a randomization procedure also frees us from fine-tuning an optimal λ value.

For the task of prediction, ML-FOREST outputs a confidence vector $\mathbf{c} = [c^1, \cdots, c^q]^\top \in \mathbb{R}^q$ for a testing example $\mathbf{x}$, where c^j represents the confidence for the j-th class. To this end, we compute the predictions of all the trees regarding $\mathbf{x}$. For each tree, we seek a decision path from the root down to a leaf node based on the prediction of classifier at each node. Basing on the relevance label vectors (i.e., $\mathbf{b}_1, \cdots, \mathbf{b}_K$) from the leaves w.r.t. all K trees, we compute the ensemble confidence outputs $\mathbf{c}$ by

$$c^j = \frac{1}{K} \sum_{k=1}^{K} b_k^j. \tag{5}$$

where b_k^j is the j-th element of the relevant label vector $\mathbf{b}_k$. More details about the prediction are given below.

3.3 PREDICTION VIA THRESHOLDING STRATEGIES

For a testing example $\mathbf{x}$, ML-FOREST outputs a prediction vector $\mathbf{y} = [y^1, \cdots, y^q]^\top$ with $y^j = 1$ indicating the j-th label is relevant regarding $\mathbf{x}$. Consider a confidence vector $\mathbf{c} = [c^1, \cdots, c^q]^\top \in \mathbb{R}^q$ for $\mathbf{x}$, where each element of $\mathbf{c}$ corresponds to a confidence value for one class label. Given $\mathbf{w}$, the prediction $\mathbf{y}$ of $\mathbf{x}$ can be completed by finding

Algorithm 3 ML-FOREST

Training Phase

Input: A training data set $\mathcal{D}$, the number of trees K

Output: A forest of tree classifiers $\mathcal{F}$

1: $\mathcal{F} = \phi$
2: **for** $i = 1$ to K **do**
3: prepare the training set $\mathcal{D}_i$ = bootstrap($\mathcal{D}$)
4: build tree classifier T_i = ML-TREE($\mathcal{D}$, none)
5: $\mathcal{F} = \mathcal{F} \cup T_i$
6: **end for**
7: **return** $\mathcal{F}$

Classification Phase

1: For a given $\mathbf{x}$, let $\mathbf{b}_1, \cdots, \mathbf{b}_K$ be the predictions assigned by the classifiers, calculate the confidence for each class, c^j, by the average combination method:

$$c^j = \frac{1}{K} \sum_{k=1}^{K} b_k^j$$

2: Assign $\mathbf{x}$ to the classes with the confidences larger than a predefined threshold value

a bipartition of relevant and irrelevant labels based on a threshold function $f_t(\mathbf{w})$ such that

$$y^j = \begin{cases} 1, & \text{if } w^j \geq t, \\ 0, & \text{otherwise.} \end{cases} \quad (6)$$

where $t \in [0, 1]$ is a predefined threshold value. There are several ways to set the threshold value t. For example, we set $t = 0.5$ for simplicity.

Besides the above simple thresholding value selection strategy, we can apply a max-drop thresholding scheme, called *Maximum Cut (MCut)* method [43], to find more flexible thresholding values for different examples automatically. In this scheme, a testing example is first assigned with a set of relevant labels. Given the confidence outputs $\mathbf{c}$ for $\mathbf{x}$, we first sort the labels according to the values of $\mathbf{c}$, and then find two adjacent classes with the largest gap/difference in terms of their confidences. Lastly, we use the mean value of these two classes as a threshold value to do prediction.

3.4 COMPUTATIONAL COMPLEXITY

Given a training data set containing m instances, a node v with a set of training data $\mathcal{D}_v$, the proposed ML-TREE algorithm uses the one-against-all paradigm to build classifiers to partition the data at each node, the complexity of this node depends on the numbers of data instances $|\mathcal{D}_v|$ and the cost to train the classifiers.

To simplify the analysis, we assume that the number of learning data in the hierarchical tree is $m = N^d$, and the tree is a complete N-ary tree with d levels, which means the average branch-out is N and all leaf nodes are at the same level [22]. Let $f(|\mathcal{D}_v|)$ be the complexity for training one binary classifier at node v. The current state-of-the-art algorithms for training SVM classifier have a time complexity scaling close to $O(m^2)$ [44]. At the root node, we have a cost of Nm^2, while at the second level we have N additional cost of $N(\frac{m}{N})^2$, i.e. an additional cost of m^2. At the next level, we have N^2 additional cost of $N(\frac{m}{N^2})^2$, i.e. an additional cost of $\frac{m^2}{N}$, and so on. Therefore, the total cost of the tree is $Nm^2(1 + \frac{1}{N} + \frac{1}{N^2} + \cdots + \frac{1}{N^d})$, which leads to $O(Nm^2)$ as a sum of a geometric series when d approaches infinity.

For the ensemble classifier including K trees, let the number of learning data in each tree be $m' = 0.632m$, and the average branch-out is N'. Thus the cost of every tree is $O(N'm'^2)$. In addition, for each tree, we have an additional cost of $O(m)$ for the sampling. Consequently, the total complexity of the ensemble classifier is $K(O(N'm'^2) + O(m))$.

Besides the complexity information of our approach, we also briefly analyze the complexities of several other well-known multi-label algorithms, i.e., binary relevance (BR) [10], classifier chain (CC) [38], and hierarchy of multi-label classifiers (HOMER) [22]. Similar to the analysis for ML-FOREST, let the time complexity for training a base classifier be $O(m^2)$ w.r.t. m training instances. BR decomposes a multi-label classification problem into q independent binary classification problems, whose overall complexity is $O(qm^2)$. CC algorithm successively trains q binary classifiers, and appends the label focused by the last classifier as a new attribute for training the next classifier. By ignoring the extended label attribute, which is usually much smaller than the dimension of the learning example, the complexity of CC is also $O(qm^2)$. For HOMER, as each training instance may pass through multiple paths from the root to leaves, it is difficult to analyze the complexity w.r.t. the number of data. In [22], it is shown that the complexity of HOMER is $O(f(q) + q)$, where q is the number of labels and $f(q)$ is the cost of the balanced clustering process used in HOMER. The running time of these algorithms are provided in the experiment section.

4 EXPERIMENTS

4.1 Experimental results on synthetic data sets

In this experiment, we first use two synthetic data sets to investigate whether our proposed ML-FOREST algorithm can generate reasonable label correlation or not. In the proposed ML-FOREST algorithm, we can use the amount of reused label co-occurrences in the leaf nodes of the tree ensemble to automatically estimate the label correlation in a multi-label data set. Specifically, we construct a m-by-q matrix $\mathbf{Q}$ where m is the number of leaf nodes in the ensemble and q is the number of possible labels. The i-th row of $\mathbf{Q}$ is equivalent to the relevant label vector $\mathbf{b}$ of the i-th leaf node, where the entries with 1 values correspond to the non-zero elements of $\mathbf{b}$. The i-th column of $\mathbf{Q}$ represents the distribution of label l_i over the leaves in the ensemble. The label correlation of two labels l_i and l_j is measured using the ϕ-coefficient defined as follows

$$\phi(i, j) = (AD - BC)/\sqrt{(A + B)(C + D)(A + C)(B + D)}, \quad (7)$$

where A, B, C and D are the frequency counts of $l_i \wedge l_j$, $l_i \wedge \neg l_j$, $\neg l_i \wedge l_j$ and $\neg l_i \wedge \neg l_j$, respectively.

As we have not given ground-truth label correlation for real-world data so far, we study two synthetic data sets [11] that we know the exact label relationship. In the experiments, we use them to check the validity of our proposed method for capturing the label correlation. The data sets have 5000 instances and five labels from l_1 to l_5. l_5

TABLE 1
Label correlation on the first synthetic data set, $l_1 = l_2$ and $l_3 = l_4$

labels	l_1	l_2	l_3	l_4	l_5
l_1	1.00	1.00	-0.14	-0.14	-0.09
l_2	1.00	1.00	-0.14	-0.14	-0.09
l_3	-0.14	-0.14	1.00	1.00	-0.14
l_4	-0.14	-0.14	1.00	1.00	-0.14
l_5	-0.09	-0.09	-0.14	-0.14	1.00

TABLE 2
Label correlation on the second synthetic data set, $l_1 = l_2 \vee l_3 \vee l_4$

labels	l_1	l_2	l_3	l_4	l_5
l_1	1.00	0.29	0.35	0.36	-0.03
l_2	0.29	1.00	0.30	0.23	-0.06
l_3	0.35	0.30	1.00	0.15	-0.03
l_4	0.36	0.23	0.15	1.00	-0.03
l_5	-0.03	-0.06	-0.03	-0.03	1.00

is assigned to an instance if it does not belong to l_1 to l_4. In the first data set $l_1 = l_2$ and $l_3 = l_4$. In the second data set $l_1 = l_2 \vee l_3 \vee l_4$. The label correlation for these two data sets are given in Table 1 and 2, respectively. The diagonals are 1.0. The last row(column) of the tables for l_5 has negative references because l_5 is assigned to an instance if it belongs to none of l_1 to l_4. Table 1 shows that the entries $(1, 2)$, $(2, 1)$, $(3, 4)$ and $(4, 3)$ are 1.0, while Table 2 shows that the entries $(1, 2)$, $(1, 3)$, $(1, 4)$ have relatively large positive values. These results are consistent with the ground-truth label correlation.

4.2 Experimental results on real data sets

In this experiment, we compare the performance of the proposed ML-FOREST algorithm with 8 well-known multi-label classification algorithms on 12 benchmark multi-label data sets, and show that the proposed algorithm is able to achieve competitive performance to the compared algorithms. For the purpose of reproducibility, we provide the code at: https://sites.google.com/site/qysite/.

4.2.1 DATA SETS

Twelve multi-label data sets are used in the experiments. These data sets are benchmark data sets from different application domains: *scene*, *emotions* and *corel5k* are image data sets, *genebase* and *yeast* are biology data sets, and the remaining seven are document corpus. Reuters(10), Reuters(21), and Reuters(90) are the Reuters-21578 text data sets w.r.t. the largest 10 classes, 21 classes, and 90 classes. All the data sets are originally split into training and test set, and we use such originally given training/test data split in the experiments [45]. The characteristics of the data sets are summarized in Table 3.

4.2.2 PARAMETER INSTANTIATION

We compare the proposed Algorithm with eight well-known multi-label classification algorithms, i.e., binary relevance (BR) [10], classifier chain (CC) [38], multi-label k nearest neighbor (ML-kNN) [18], instance-based and logistic regression (IBLR-ML) [21], hierarchy of multi-label classifiers (HOMER) [22] and predictive clustering trees (PCT) [32], random forest of predictive clustering trees (RF-PCT) [35] and Two stage architecture (TSA) [36]. The implementations of the BR, CC, ML-kNN, IBLR-ML, HOMER, RF-PCT and TSA algorithms are based on the MULAN library[1] and the implementation of the PCT algorithm is based on the CLUS system[2].

TABLE 3
Description of the multi-label data sets in terms of the number of training (#tr.e.) and test (#t.e.) examples, the number of features (f), the total number of labels(q) and the label cardinality (l_c). The data sets are ordered by their label cardinality (i.e., average number of labels per example)

data set	domain	#tr.e.	#t.e.	f	q	l_c
scene	Image	1,211	1,196	294	6	1.07
Reuters(10)	Text	6,490	2,545	500	10	1.11
Reuters(21)	Text	7,140	2,747	500	21	1.16
Reuters(90)	Text	7,770	3,019	500	90	1.24
medical	Text	333	645	1,449	45	1.25
genebase	Biology	463	199	1,186	27	1.25
ohsumed	Text	6,286	7,643	500	24	1.66
emotions	Image	391	202	72	6	1.87
tmc2007	Text	21,519	7,077	500	22	2.16
bibtex	Text	4,880	2,515	1,836	159	2.40
corel5k	Image	4,500	500	499	374	3.52
yeast	Biology	1,500	917	103	14	4.2

For the algorithms using base classifiers (i.e., BR, CC, HOMER, TSA and ML-FOREST), SVM with linear kernel in LIBSVM library [46] is used as the base classifier. The LIBSVM with options "-b 1" is used to learn SVMs with probability outputs in the experiments. For parameter tuning, we use 5-fold cross validation on the training set to select parameters in the experiments. In particular, for each algorithm on each data set, the parameters yield the best average hamming loss using cross validation are selected. Then, the algorithm is trained again with the selected parameters on the whole training set and evaluated on the test set for comparison. The parameter tunings of different algorithms are given as follows. The number of neighbors k in the ML-kNN and IBLR-ML is tuned from 5 to 30 with an increment of 5. The number of clusters in the HOMER is tuned within the range of 2 to 6. The values considered for parameter C of SVM for BR, CC, TSA and HOMER are tuned with the values $2^{-5}, 2^{-3}, ..., 2^3$. For RF-PCT, we set the number of models to 50. We try the feature subset sizes of $\log_2 f + 1$, $\sqrt{f}$, $f/10$ and $f/2$, where f is the total number of features. Such setting is similar to the testing in [47], and we find that $f/2$ results in best performance, thus we use $f/2$ as the feature subset size in the model.

Our ML-FOREST algorithm has three essential parameters: the number of trees K, the purity threshold λ and the penalty parameter C of the SVM base classifiers. The parameter λ does not need to be extensively tuned. For each tree, we randomly selected λ in the range $(0.9, 0.95)$. We also simply set $K = 50$ and $C = 2^{-5}$ as default. The parameter setting will be discussed in the later section. We set the number of trees $K = 50$ as the same in [48], and we set $C = 2^{-5}$ because it is usually used as default setting for SVM [49]. Unless otherwise stated, we use these default settings in the experiments.

1. http://mulan.sourceforge.net
2. http://clus.sourceforge.net

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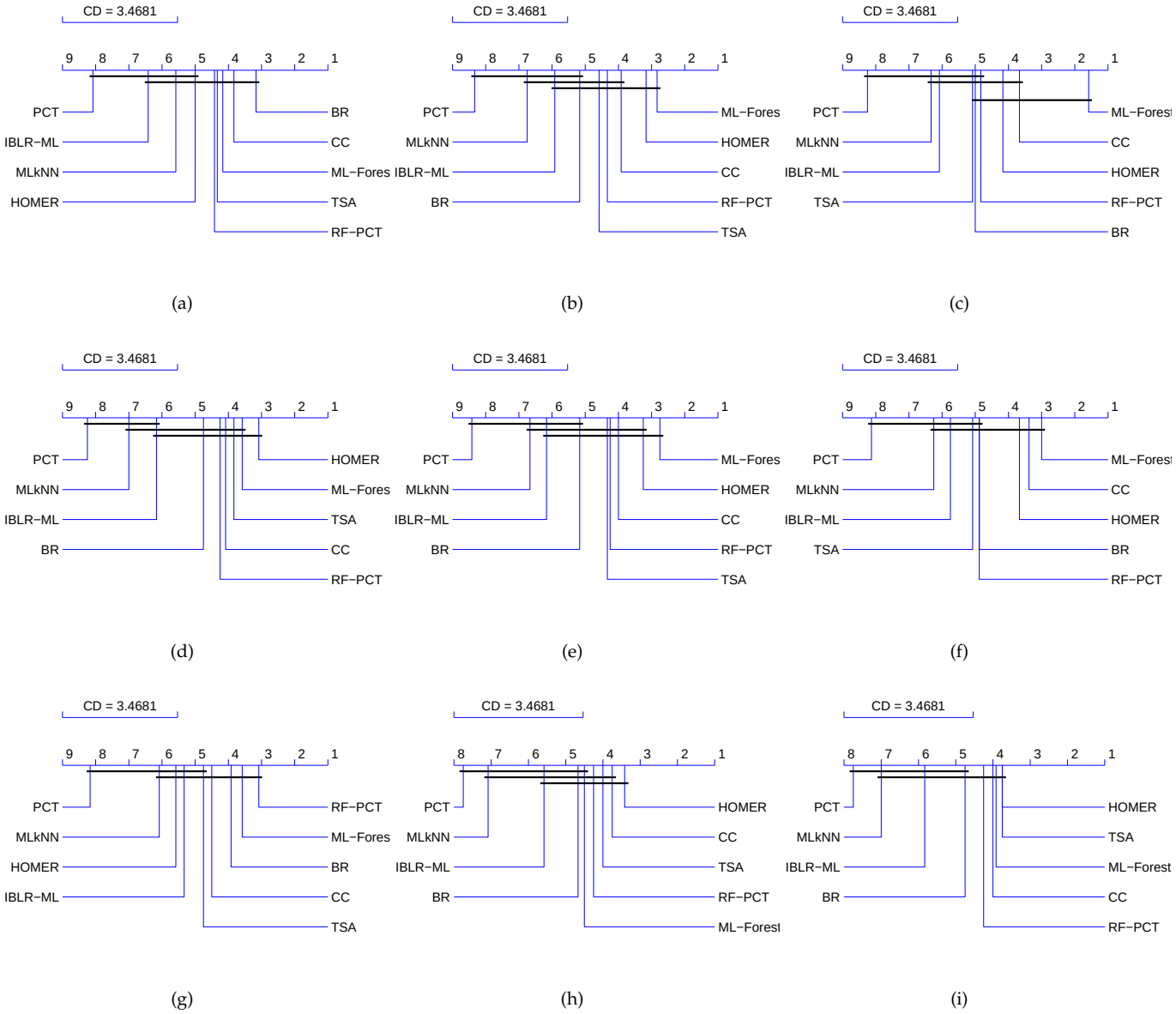

Fig. 3. The average ranks diagrams for the bipartition-based measures: (a) hamming loss, (b) accuracy, (c) precision, (d) recall, (e) F1-score, (f) subset accuracy, (g) macro-precision, (h) macro-recall, (i) macro-F1.

4.2.3 Performance Measures

The performance of multi-label classification is measured by the *bipartition-based* metrics which are based on the comparison of the predicted labels of each example with the ground-truth labels provided by the data set. In our experiments, nine *bipartition-based* metrics (hamming loss, example-based accuracy, example-based precision, example-based recall, example-based F1, subset accuracy, macro-precision, macro-recall and macro-F1) are used to measure the performance. Please see [50] the detailed definitions of these metrics. On the other hand, the ranking-based metrics (e.g., one-error, coverage, ranking loss and average precision) compare the predicted ranking of the labels with the ground truth ranking.

We also consider four ranking-based metrics (one-error, coverage, ranking loss, and average precision) for multi-label ranking evaluation. Ranking-based metrics are threshold independent. These measures compare the predicted ranking of the labels with the ground truth ranking.

As multiple algorithms are compared on multiple data sets in the experiments, we follow a two-step statistical test procedure (the corrected Friedman test and the post-hoc Nemenyi test) as recommended by Demšar [16] to compare the algorithms in a pairwise way across multiple data sets. The comparison results of the algorithms w.r.t. different evaluation metrics are given in Tables 4 to 17. The Friedman test is a non-parametric test for multiple hypotheses testing. The procedure involves ranking the algorithms (each row in each section of the table) in a descending order based on their performance for each data set separately in which the best performance for each data set gets the rank of 1.

The average rank of each algorithm across all the data sets (each column in each section of the table) is computed. Then, the Nemenyi post-hoc test is used in order to detect

which algorithms are significantly different from each other, based upon the average ranks of the algorithms. The performances of two algorithms is significantly different if their average ranks differ by more than a critical distance (CD) whose value is depended on the number of algorithms, the number of data sets and a given significance level p. The results of the Nemenyi post-hoc test can be visually presented with diagrams [16] as shown in Fig. 3, where the critical distance is 2.849 and the significant level is $p = 0.05$. For each evaluation metric, the values on the horizontal axis show the average ranks of the algorithms in a manner that the algorithm has the best (worst) rank which is at the rightmost (left-most) side of the diagram. The algorithms that do not differ significantly are connected by a bold horizontal line.

4.2.4 Results on the bipartition-based metrics

Fig. 3 shows the presentations of the results on the multi-label classification data sets for the *bipartition-based* metrics. Tables 4 to 12 show the complete comparison results of the algorithms for different evaluation metrics. The table is separated into several sections, each of them representing a specific evaluation metric. For each evaluation metric, the uparrow $\uparrow$ (downarrow $\downarrow$) indicates that a larger (smaller) value is more useful for such a specific evaluation metric. The numbers in brackets are the ranks of the algorithms in terms of one particular evaluation metric, and the numbers in boldface indicate the best ranking algorithms. The last row in each section of the table is the average ranks of the algorithms across all the data sets. According to the experimental results, ML-FOREST is able to achieve competitive performance with the compared algorithms, and the experiments reveal a number of interesting points:

- The ML-FOREST, TSA, RF-PCT and HOMER methods are hierarchical methods using different strategies to exploit the label dependency. ML-FOREST, TSA and HOMER are based on problem transformation mechanism using SVM base classifiers to solve a hierarchy of partial binary classification problems, whilst RF-PCT is to utilize multiple component classifiers each deals with a partial data set. We observe that this type of methods consistently achieve better performance than the other methods, such as MLkNN and PCT, though the *bipartition-based* evaluation metrics that are used. MLkNN and PCT are not competitive mainly due to the inadequacy of modeling label dependency. This suggests the importance of leveraging label dependency for multi-label classification.
- Comparing ML-FOREST and HOMER, we see that both ML-FOREST and HOMER has excellent overall performances. HOMER ranks 1st in terms of *recall* and *macro-recall*, While ML-FOREST is better than HOMER in terms of *precision*. Precision and recall are two different quantitative measures evaluate the algorithm performance from different aspects. Precision is the fraction of predicted labels that are also relevant (evaluating the accurate of the prediction), while recall is the fraction of relevant labels that are predicted correctly (evaluating the completeness of the prediction). This result indicates that the predictions from ML-FOREST are more accurate than those from HOMER, while the predictions from HOMER are more complete than the ones from ML-FOREST. A reasonable explanation of this finding is that, HOMER uses multiple-leaf labeling method to classify a new multi-label example. Homer returns the union of the predicted label set in multiple leaves, therefore the prediction is more complete. Whilst ML-FOREST is based on decision-path labeling method which combines the predictive labels from the root to a leaf as prediction result. The prediction at each node is obtained by the purity threshold λ with a large value in our experiments and therefore, is more accurate.
- Comparing ML-FOREST and RF-PCT, one observes that both ML-FOREST and RF-PCT consistently yield good, though perhaps not the best, performance in terms of all the evaluation metrics. ML-FOREST and RF-PCT perform well in terms of *macro precision* and *macro recall* which take the average of the precision and recall over different classes. This result implies that these two tree ensemble methods are robust across a range of different datasets and classes. Both of them are able to achieve good overall performance in the evaluation. The reason for this is that although growing each individual tree in ML-FOREST and RF-PCT may not be optimal, multiple fully grown trees can make it up for good and robust performance.
- We further analyze the performance of the ML-FOREST w.r.t. different types of data. We can see that the performances of ML-FOREST on text data sets (e.g., Reuters, ohsumed and bibtex) are better than those of the image and biology data sets. A reasonable explanation for this finding is that we use SVMs with a linear kernel as base classifier for ML-FOREST. Previous study [49] has shown that most text categorization problems are linearly separable and so linear kernel is often applied for text categorization tasks, while a radial basis kernel is more appropriate for image and biology data. In practice, the prior knowledge of which kernel function should be used is difficult to obtain in advance.

4.2.5 Results on the ranking-based metrics

In this subsection, we present results w.r.t ranking evaluation metrics. Fig. 4 shows the graphical presentation of the results on the multi-label classification data sets for the ranking-based metrics. Tables 13 to 16 show the complete comparison results of the algorithms for these metrics.

The best performing methods are TSA and BR, followed by RF-PCT and ML-Forest. RF-PCT is robust across both bipartition-based metrics and ranking-based metrics. On the other hand, even though the HOMER approach is able to produce good results for the example-based measures, it performs poorly across all the ranking-based evaluation measures. A similar observation is also found in recent studies [50], where it is shown that even sophisticate approaches are not able to outperform all other methods in all the measures. The evaluation measures used in the experiments assess the learning performance from different aspects and

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TABLE 4
The performance of the multi-label classification algorithms in terms of **hamming loss**↓.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.1028(5)	0.1062(7)	0.0960(2)	**0.0893(1)**	0.1180(8)	0.1332(9)	0.0977(4)	0.1041(6)	0.0966(3)
Reuters(10)	0.0165(3)	0.0166(4)	0.0218(8)	0.0208(7)	**0.0155(1)**	0.0288(9)	0.0187(6)	0.0160(2)	0.0183(5)
Reuters(21)	0.0110(2)	**0.0109(1)**	0.0141(8)	0.0130(7)	0.0115(4)	0.0214(9)	0.0129(6)	0.0111(3)	0.0128(5)
Reuters(90)	0.0050(2)	**0.0049(1)**	0.0054(6)	0.0058(8)	0.0053(5)	0.0118(9)	0.0057(7)	0.0051(3)	0.0052(4)
medical	**0.0119(1)**	0.0120(2)	0.0197(7)	0.0299(9)	0.0141(6)	0.0230(8)	0.0130(4)	0.0131(5)	0.0121(3)
genbase	0.0746(9)	0.0744(8)	0.0015(3)	0.0016(4)	0.0030(5)	0.0078(7)	**0.0007(1)**	0.0011(2)	0.0037(6)
ohsumed_norm	0.0434(2)	**0.0431(1)**	0.0564(5)	0.0569(6)	0.0480(4)	0.0624(9)	0.0595(8)	0.0580(7)	0.0450(3)
emotions	0.2137(3)	0.2459(4)	0.2830(9)	0.2797(8)	0.2112(2)	0.2591(6)	**0.1972(1)**	0.2632(7)	0.2564(5)
tmc2007	0.0553(3)	0.0554(4)	0.0611(8)	0.0583(6)	0.0578(5)	0.0759(9)	**0.0495(1)**	0.0550(2)	0.0601(7)
bibtex	0.0123(3)	0.0124(4)	0.0139(7)	0.0189(9)	0.0137(6)	0.0145(8)	0.0130(5)	0.0122(2)	**0.0120(1)**
corel5k	**0.0092(1)**	0.0093(2)	0.0094(3)	0.0231(9)	0.0103(7)	0.0096(5)	0.0095(4)	0.0119(8)	0.0100(6)
yeast	0.2048(4)	0.2196(8)	**0.1980(1)**	0.2005(3)	0.2119(7)	0.2426(9)	0.2065(6)	0.2060(5)	0.1985(2)
Avg. rank	**3.17**	3.83	5.58	6.42	5.00	8.08	4.42	4.33	4.17

TABLE 5
The performance of the multi-label classification algorithms in terms of **accuracy** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.5811(7)	0.6702(2)	0.6240(5)	0.6495(4)	0.5838(6)	0.5698(9)	0.6608(3)	0.5811(8)	**0.7189(1)**
Reuters(10)	0.8945(6)	0.9038(3)	0.8600(9)	0.8706(7)	0.9054(2)	0.8602(8)	0.8991(5)	0.8994(4)	**0.9153(1)**
Reuters(21)	0.8689(4)	0.8796(2)	0.8553(6)	0.8511(8)	0.8731(3)	0.7794(9)	0.8537(7)	0.8684(5)	**0.8837(1)**
Reuters(90)	0.7741(6)	0.7874(3)	0.7788(5)	0.7860(4)	**0.7972(1)**	0.5678(9)	0.7295(8)	0.7738(7)	0.7932(2)
medical	0.7048(3)	0.7074(2)	0.3731(8)	0.4225(7)	0.6712(5)	0.2036(9)	0.6740(4)	0.6488(6)	**0.7590(1)**
genbase	0.3595(9)	0.3600(8)	0.9782(4)	0.9849(3)	0.9652(6)	0.9284(7)	**0.9916(1)**	0.9866(2)	0.9673(5)
ohsumed	0.5028(4)	0.5219(3)	0.3241(6)	0.2761(7)	0.5290(2)	0.1878(9)	0.2349(8)	0.3649(5)	**0.5571(1)**
emotions	0.5173(2)	0.5035(3)	0.3177(8)	0.3160(9)	0.4917(4)	0.4827(5)	**0.5804(1)**	0.4344(6)	0.4287(7)
tmc2007	0.5953(5)	0.5987(4)	0.5567(7)	0.5651(6)	0.6024(2)	0.4544(9)	**0.6393(1)**	0.5997(3)	0.5243(8)
bibtex	0.2865(5)	0.3067(4)	0.1294(8)	0.1651(7)	0.3352(2)	0.0462(9)	0.2299(6)	0.3179(3)	**0.3568(1)**
corel5k	0.0501(5)	0.0553(4)	0.0184(8)	0.0412(6)	0.1055(3)	0.0000(9)	0.0222(7)	**0.1934(1)**	0.1194(2)
yeast	0.4984(6)	0.4352(9)	0.4920(7)	0.5031(3)	0.5195(2)	0.4530(8)	**0.5341(1)**	0.4991(5)	0.5009(4)
Avg. rank	5.17	3.92	6.75	5.92	3.17	8.33	4.33	4.58	**2.83**

TABLE 6
The performance of the multi-label classification algorithms in terms of **precision** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.6063(7)	0.7032(2)	0.6547(5)	0.6798(4)	0.6127(6)	0.5991(9)	0.6898(3)	0.6060(8)	**0.7573(1)**
Reuters(10)	0.9075(6)	0.9158(3)	0.8726(8)	0.8819(7)	0.9168(2)	0.8725(9)	0.9155(4)	0.9104(5)	**0.9409(1)**
Reuters(21)	0.8890(4)	0.8989(2)	0.8766(6)	0.8713(8)	0.8905(3)	0.8129(9)	0.8732(7)	0.8870(5)	**0.9340(1)**
Reuters(90)	0.8045(6)	0.8179(3)	0.8111(5)	0.8163(4)	0.8245(2)	0.5790(9)	0.7607(8)	0.8018(7)	**0.8524(1)**
medical	0.7450(3)	0.7470(2)	0.4163(8)	0.4668(7)	0.7183(5)	0.2574(9)	0.7276(4)	0.6875(6)	**0.8451(1)**
genbase	0.3613(9)	0.3617(8)	0.9899(6)	0.9975(2)	0.9807(7)	0.9950(3)	**1.0000(1)**	0.9950(4)	0.9941(5)
ohsumed	0.6373(4)	0.6609(2)	0.4339(6)	0.3708(7)	0.6473(3)	0.2656(9)	0.3387(8)	0.4646(5)	**0.7785(1)**
emotions	0.6229(4)	0.6007(5)	0.5107(8)	0.4942(9)	0.6345(3)	0.5817(6)	**0.6980(1)**	0.5489(7)	0.6619(2)
tmc2007	0.7607(4)	0.7627(3)	0.7243(8)	0.7283(7)	0.7337(6)	0.6481(9)	0.7699(2)	0.7603(5)	**0.7779(1)**
bibtex	0.4575(5)	0.4679(4)	0.2543(7)	0.2531(8)	0.4767(3)	0.1396(9)	0.3839(6)	0.4832(2)	**0.5656(1)**
corel5k	0.1298(4)	0.1157(5)	0.0425(8)	0.0680(7)	0.2146(3)	0.0000(9)	0.0770(6)	**0.3242(1)**	0.2680(2)
yeast	0.6994(4)	0.6966(5)	**0.7322(1)**	0.7069(3)	0.6686(7)	0.6370(9)	0.6678(8)	0.6941(6)	0.7171(2)
Avg. rank	5.00	3.67	6.33	6.08	4.17	8.25	4.83	5.08	**1.58**

TABLE 7
The performance of the multi-label classification algorithms in terms of **recall** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.6217(8)	0.6877(3)	0.6488(5)	0.6576(4)	0.6346(6)	0.5711(9)	0.6948(2)	0.6325(7)	**0.7441(1)**
Reuters(10)	0.9111(6)	0.9183(5)	0.8750(8)	0.8849(7)	**0.9244(1)**	0.8714(9)	0.9200(4)	0.9213(3)	0.9227(2)
Reuters(21)	0.8897(5)	0.8993(2)	0.8743(6)	0.8668(8)	0.8968(3)	0.7816(9)	0.8710(7)	0.8933(4)	**0.9093(1)**
Reuters(90)	0.7942(7)	0.8056(2)	0.7963(5)	0.8027(4)	**0.8227(1)**	0.5683(9)	0.7383(8)	0.7963(6)	0.8051(3)
medical	0.7628(2)	0.7509(3)	0.3953(8)	0.5339(7)	0.7186(4)	0.2036(9)	0.7116(5)	0.6997(6)	**0.7822(1)**
genbase	0.9891(3)	0.9890(4)	0.9782(7)	0.9874(5)	0.9795(6)	0.9284(9)	**0.9916(1)**	0.9915(2)	0.9732(8)
ohsumed_norm	0.5479(4)	0.5679(3)	0.3579(6)	0.2961(7)	**0.6234(1)**	0.1878(9)	0.2417(8)	0.4484(5)	0.5681(2)
emotions	0.6089(2)	0.5965(4)	0.3639(8)	0.3622(9)	0.5371(6)	0.5842(5)	**0.7005(1)**	0.5998(3)	0.4893(7)
tmc2007	0.6940(4)	0.6930(5)	0.6542(7)	0.6546(6)	0.7383(2)	0.5322(9)	**0.7477(1)**	0.7036(3)	0.5334(8)
bibtex	0.2949(5)	0.3199(4)	0.1335(8)	0.2095(7)	0.3800(2)	0.0462(9)	0.2453(6)	0.3393(3)	**0.3952(1)**
corel5k	0.0515(6)	0.0587(5)	0.0190(8)	0.0772(4)	0.1225(3)	0.0000(9)	0.0223(7)	**0.2625(1)**	0.1423(2)
yeast	0.5822(5)	0.4950(9)	0.5491(8)	0.5783(6)	0.6327(2)	0.5860(4)	**0.6749(1)**	0.5903(3)	0.5730(7)
Avg. rank	4.75	4.08	7.00	6.17	**3.08**	8.25	4.25	3.83	3.58

TABLE 8
The performance of the multi-label classification algorithms in terms of **F1 score** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.6030(8)	0.6870(2)	0.6426(5)	0.6623(4)	0.6102(6)	0.5800(9)	0.6819(3)	0.6062(7)	**0.7400(1)**
Reuters(10)	0.9045(6)	0.9128(3)	0.8694(8)	0.8792(7)	0.9156(2)	0.8682(9)	0.9115(4)	0.9103(5)	**0.9261(1)**
Reuters(21)	0.8826(5)	0.8927(2)	0.8691(6)	0.8631(8)	0.8870(3)	0.7913(9)	0.8661(7)	0.8828(4)	**0.9013(1)**
Reuters(90)	0.7907(6)	0.8035(3)	0.7949(5)	0.8012(4)	0.8148(2)	0.5712(9)	0.7421(8)	0.7902(7)	**0.8151(1)**
medical	0.7380(2)	0.7361(3)	0.3951(8)	0.4733(7)	0.7040(5)	0.2209(9)	0.7043(4)	0.6793(6)	**0.7945(1)**
genbase	0.5235(9)	0.5240(8)	0.9821(4)	0.9900(3)	0.9749(6)	0.9467(7)	**0.9941(1)**	0.9908(2)	0.9769(5)
ohsumed	0.5615(4)	0.5824(3)	0.3703(6)	0.3125(7)	0.6018(2)	0.2107(9)	0.2679(8)	0.4260(5)	**0.6265(1)**
emotions	0.5901(2)	0.5766(3)	0.3959(8)	0.3911(9)	0.5576(4)	0.5559(5)	**0.6675(1)**	0.5323(6)	0.5236(7)
tmc2007	0.6863(5)	0.6867(4)	0.6475(7)	0.6513(6)	0.6961(2)	0.5444(9)	**0.7265(1)**	0.6906(3)	0.6052(8)
bibtex	0.3343(5)	0.3559(4)	0.1619(8)	0.2087(7)	0.3944(2)	0.0671(9)	0.2764(6)	0.3718(3)	**0.4270(1)**
corel5k	0.0704(5)	0.0733(4)	0.0249(8)	0.0606(6)	0.1436(3)	0.0000(9)	0.0344(7)	**0.2774(1)**	0.1722(2)
yeast	0.6087(5)	0.5420(9)	0.5993(7)	0.6085(6)	0.6243(2)	0.5668(8)	**0.6459(1)**	0.6103(3)	0.6090(4)
Avg. rank	5.17	4.00	6.67	6.17	3.25	8.42	4.25	4.33	**2.75**

TABLE 9
The performance of the multi-label classification algorithms in terms of **subset accuracy** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.5159(7)	0.6196(2)	0.5686(5)	0.6112(3)	0.5050(9)	0.5393(6)	0.5978(4)	0.5075(8)	**0.6564(1)**
Reuters(10)	0.8633(5)	0.8758(2)	0.8295(9)	0.8428(7)	0.8739(3)	0.8354(8)	0.8613(6)	0.8656(4)	**0.8833(1)**
Reuters(21)	0.8260(4)	**0.8391(1)**	0.8133(7)	0.8129(8)	0.8296(3)	0.7426(9)	0.8147(6)	0.8231(5)	0.8333(2)
Reuters(90)	0.7237(6)	0.7393(3)	0.7307(5)	0.7406(2)	**0.7436(1)**	0.5588(9)	0.6916(8)	0.7235(7)	0.7317(4)
medical	0.6078(3)	0.6248(2)	0.3070(7)	0.2853(8)	0.5736(5)	0.1535(9)	0.5829(4)	0.5581(6)	**0.6543(1)**
genbase	0.0000(8.5)	0.0000(8.5)	0.9648(3)	0.9648(4)	0.9296(6)	0.8844(7)	**0.9849(1)**	0.9749(2)	0.9397(5)
ohsumed_norm	0.3318(3)	0.3463(2)	0.1973(5)	0.1748(7)	0.3187(4)	0.1267(9)	0.1468(8)	0.1934(6)	**0.3718(1)**
emotions	0.2921(2)	0.2772(3)	0.1040(8)	0.1040(9)	0.2772(4)	0.2624(5)	**0.3119(1)**	0.1287(7)	0.1683(6)
tmc2007	0.3144(4)	0.3277(2)	0.2816(7)	0.2960(6)	0.3054(5)	0.2025(9)	**0.3534(1)**	0.3164(3)	0.2778(8)
bibtex	0.1678(5)	0.1817(3)	0.0549(8)	0.0708(7)	0.1825(2)	0.0036(9)	0.1125(6)	0.1765(4)	**0.2073(1)**
corel5k	0.0040(5)	0.0060(4)	0.0001(7)	0.0020(6)	**0.0180(1)**	0.0000(8.5)	0.0000(8.5)	0.0120(2)	0.0100(3)
yeast	0.1516(6)	0.1396(8)	0.1592(4)	0.1778(2)	**0.1963(1)**	0.1178(9)	0.1559(5)	0.1407(7)	0.1658(3)
Avg. rank	4.83	3.42	6.25	5.75	3.67	8.13	4.88	5.08	**3.00**

TABLE 10
The performance of the multi-label classification algorithms in terms of **macro-precision** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.7781(4)	0.7453(7)	0.7931(3)	**0.8314(1)**	0.6943(8)	0.6561(9)	0.7970(2)	0.7641(5)	0.7486(6)
Reuters(10)	**0.9123(1)**	0.9083(2)	0.8740(8)	0.8812(7)	0.9014(3)	0.8250(9)	0.8916(5)	0.9007(4)	0.8868(6)
Reuters(21)	0.8690(3)	0.8605(4)	0.8230(8)	0.8584(5)	0.8291(7)	0.6492(9)	**0.9202(1)**	0.8572(6)	0.8895(2)
Reuters(90)	0.5723(4)	**0.5952(1)**	0.5429(5)	0.4503(7)	0.5771(3)	0.0337(9)	0.4291(8)	0.5344(6)	0.5942(2)
medical	0.3372(4)	0.3571(3)	0.1477(8)	0.2023(7)	0.3130(5)	0.0185(9)	**0.3810(1)**	0.2665(6)	0.3665(2)
genbase	0.6994(7)	0.7041(4)	0.7037(5)	0.7391(2)	0.6139(8)	0.4059(9)	**0.8519(1)**	0.7325(3)	0.7037(6)
ohsumed	0.7206(2)	0.7077(3)	0.6331(5)	0.6524(4)	0.6288(6)	0.1189(9)	0.5935(8)	0.6016(7)	**0.7898(1)**
emotions	0.6842(3)	0.5956(7)	0.4751(9)	0.4773(8)	0.7051(2)	0.6194(4)	**0.7165(1)**	0.6006(6)	0.6043(5)
tmc2007	0.7812(5)	0.7664(6)	0.7376(7)	0.7828(4)	0.6951(8)	0.3948(9)	0.8458(2)	0.8042(3)	**0.9237(1)**
bibtex	0.5220(3)	0.5027(4)	0.1941(7)	0.1698(8)	0.4311(6)	0.0063(9)	0.4410(5)	0.5301(2)	**0.6249(1)**
corel5k	0.0516(5)	0.0523(4)	0.0334(7)	0.0329(8)	0.0553(3)	0.0000(9)	**0.3105(1)**	0.0616(2)	0.0516(6)
yeast	0.3729(6)	0.3231(9)	**0.6003(1)**	0.5101(3)	0.3460(8)	0.4099(4)	0.5626(2)	0.3677(7)	0.3932(5)
Avg. rank	3.92	4.50	6.08	5.33	5.58	8.17	**3.08**	4.75	3.58

TABLE 11
The performance of the multi-label classification algorithms in terms of **macro-recall** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.6134(8)	0.6770(3)	0.6374(5)	0.6442(4)	0.6270(6)	0.5605(9)	0.6792(2)	0.6250(7)	**0.7297(1)**
Reuters(10)	0.8237(4)	0.8368(3)	0.7539(8)	0.7415(9)	**0.8571(1)**	0.8062(5)	0.7886(6)	0.8487(2)	0.7769(7)
Reuters(21)	0.7460(4)	**0.7579(1)**	0.6980(5)	0.6952(6)	0.7564(3)	0.5442(9)	0.6171(8)	0.7577(2)	0.6875(7)
Reuters(90)	0.3983(4)	0.4014(3)	0.3551(7)	0.4058(2)	**0.4427(1)**	0.0375(9)	0.2161(8)	0.3772(6)	0.3809(5)
medical	0.3066(4)	0.3191(3)	0.0859(8)	0.2141(7)	0.2557(5)	0.0220(9)	**0.3498(1)**	0.2353(6)	0.3266(2)
genbase	0.7755(2)	0.7755(3)	0.6961(6)	0.7189(5)	0.6366(8)	0.4021(9)	**0.8519(1)**	0.7407(4)	0.6479(7)
ohsumed	0.4100(4)	0.4383(2)	0.2365(6)	0.2087(7)	**0.5308(1)**	0.0822(9)	0.1403(8)	0.3148(5)	0.4276(3)
emotions	0.6048(3)	0.5880(4)	0.3129(9)	0.3284(8)	0.5286(6)	0.5853(5)	**0.6824(1)**	0.6131(2)	0.4772(7)
tmc2007	0.4878(5)	0.5006(4)	0.4073(7)	0.4269(6)	0.5614(2)	0.2601(9)	0.3945(8)	0.5021(3)	**0.6275(1)**
bibtex	0.1698(5)	0.1922(4)	0.0508(8)	0.1177(7)	**0.2600(1)**	0.0063(9)	0.1366(6)	0.2098(3)	0.2372(2)
corel5k	0.0135(7)	0.0216(6)	0.0090(8)	0.0428(2)	0.0307(4)	0.0000(9)	**0.2995(1)**	0.0387(3)	0.0298(5)
yeast	0.3233(6)	0.2705(9)	0.3075(8)	0.3379(4)	0.3682(3)	0.3842(2)	**0.3980(1)**	0.3333(5)	0.3195(7)
Avg. rank	4.67	3.75	7.08	5.58	**3.42**	7.75	4.25	4.00	4.50

TABLE 12
The performance of the multi-label classification algorithms in terms of **macro-F1** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.6853(7)	0.7063(4)	0.7003(5)	0.7226(3)	0.6575(8)	0.6025(9)	0.7322(2)	0.6865(6)	**0.7375(1)**
Reuters(10)	0.8635(4)	0.8692(3)	0.8050(8)	0.7998(9)	**0.8769(1)**	0.8132(7)	0.8338(5)	0.8720(2)	0.8239(6)
Reuters(21)	0.7996(3)	**0.8028(1)**	0.7408(7)	0.7612(6)	0.7876(4)	0.5576(9)	0.7112(8)	0.8005(2)	0.7780(5)
Reuters(90)	0.4503(3)	0.4574(2)	0.4077(7)	0.4090(6)	**0.4749(1)**	0.0336(9)	0.2630(8)	0.4256(5)	0.4403(4)
medical	0.3124(4)	0.3271(3)	0.1037(8)	0.1815(7)	0.2687(5)	0.0201(9)	**0.3581(1)**	0.2431(6)	0.3358(2)
genbase	0.7008(5)	0.7032(4)	0.6998(6)	0.7258(3)	0.6183(8)	0.4038(9)	**0.8519(1)**	0.7361(2)	0.6718(7)
ohsumed	0.5023(4)	0.5267(2)	0.3174(6)	0.3046(7)	**0.5437(1)**	0.0958(9)	0.1753(8)	0.3934(5)	0.5227(3)
emotions	0.6072(2)	0.5568(6)	0.3469(9)	0.3705(8)	0.5781(5)	0.5957(4)	**0.6800(1)**	0.6027(3)	0.5249(7)
tmc2007	0.5691(5)	0.5803(4)	0.4850(7)	0.5212(6)	0.6051(2)	0.2938(9)	0.4460(8)	0.5813(3)	**0.7009(1)**
bibtex	0.2241(5)	0.2470(4)	0.0692(8)	0.1308(7)	0.3007(2)	0.0063(9)	0.1834(6)	0.2678(3)	**0.3129(1)**
corel5k	0.0185(7)	0.0263(6)	0.0128(8)	0.0295(5)	0.0333(4)	0.0000(9)	**0.3008(1)**	0.0412(2)	0.0345(3)
yeast	0.3247(8)	0.2785(9)	0.3361(5)	0.3651(3)	0.3467(4)	**0.3914(1)**	0.3784(2)	0.3354(6)	0.3332(7)
Avg. rank	4.75	4.00	7.00	5.83	3.75	7.75	4.25	**3.75**	3.92

TABLE 13
The performance of the multi-label classification algorithms in terms of **one-error** ↓.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.2425(3)	0.2776(7)	0.2533(6)	**0.2341(1)**	0.3294(8)	0.3813(9)	0.2475(4)	0.2366(2)	0.2475(5)
Reuters(10)	0.0530(2)	0.0534(4)	0.0896(8)	0.0864(7)	0.0727(6)	0.1218(9)	0.0625(5)	**0.0515(1)**	0.0531(3)
Reuters(21)	0.0641(2)	0.0648(3)	0.1012(8)	0.0910(6)	0.0961(7)	0.1744(9)	0.0823(5)	**0.0593(1)**	0.0673(4)
Reuters(90)	0.1447(3)	0.1414(2)	0.1534(5)	0.1613(6)	0.1650(8)	0.4193(9)	0.1630(7)	**0.1395(1)**	0.1467(4)
medical	0.1597(2)	0.1690(3)	0.3612(7)	0.5256(8)	0.2465(6)	0.6279(9)	0.2016(5)	0.1721(4)	**0.1504(1)**
genbase	**0.0000(2.5)**	0.0000(2.5)	0.0000(2.5)	0.0000(2.5)	0.0050(7)	0.0050(8)	0.0000(5)	0.0000(6)	0.0050(9)
ohsumed_norm	0.2281(3)	**0.2224(1)**	0.3892(6)	0.4117(7)	0.3202(4)	0.5393(9)	0.4438(8)	0.3598(5)	0.2239(2)
emotions	**0.2921(1)**	0.3614(7)	0.3615(8)	0.3564(6)	0.3515(5)	0.3812(9)	0.2970(2)	0.3020(3)	0.3020(4)
tmc2007	0.1710(4)	0.1749(5)	0.2040(7)	0.1946(6)	0.2374(8)	0.2984(9)	0.1542(2)	0.1600(3)	**0.1170(1)**
bibtex	0.3575(2)	0.3586(3)	0.6020(7)	0.6294(8)	0.4584(6)	0.7829(9)	0.4123(5)	0.3718(4)	**0.3458(1)**
corel5k	0.6760(3)	0.6860(4)	0.6940(5)	0.8820(9)	0.7580(7)	0.7680(8)	0.6400(2)	**0.5980(1)**	0.7140(6)
yeast	0.2443(7)	0.3162(9)	0.2345(2)	0.2410(4)	0.2497(8)	**0.2334(1)**	0.2443(6)	0.2356(3)	0.2410(5)
Avg. rank	**3.04**	4.21	5.96	5.88	6.67	8.17	4.67	2.83	3.75

TABLE 14
The performance of the multi-label classification algorithms in terms of **coverage** ↓.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.5017(2)	0.5761(6)	0.5326(4)	0.5485(5)	1.1204(9)	0.9607(8)	0.5284(3)	**0.4900(1)**	0.6806(7)
Reuters(10)	0.2016(2)	0.2059(3)	0.3434(7)	0.3324(6)	0.5839(9)	0.3937(8)	0.2122(4)	**0.1737(1)**	0.3136(5)
Reuters(21)	0.3611(2)	0.3648(3)	0.7197(7)	0.5643(5)	1.6702(9)	0.9425(8)	0.3702(4)	**0.3189(1)**	0.6309(6)
Reuters(90)	2.0768(3)	2.1007(4)	2.9248(5)	3.1977(7)	12.7410(9)	7.1332(8)	1.7857(2)	**1.4495(1)**	3.0113(6)
medical	**2.1349(1)**	2.2977(3)	3.2775(6)	5.5488(7)	5.9922(9)	5.8977(8)	2.1457(2)	2.3752(4)	3.2000(5)
genbase	0.4925(3)	0.4925(4)	0.5678(8)	0.5025(5)	0.5226(6)	**0.3116(1)**	0.5477(7)	0.5779(9)	0.4774(2)
ohsumed	2.6219(2)	**2.4900(1)**	4.1917(4)	4.6049(7)	9.8917(9)	6.6931(8)	4.3671(5)	3.3690(3)	4.3869(6)
emotions	1.9455(2)	1.9851(4)	2.5149(8)	2.2970(6)	2.6634(9)	2.3515(7)	**1.9208(1)**	1.9505(3)	2.0545(5)
tmc2007	2.4972(6)	2.5796(7)	2.4054(5)	2.3213(4)	7.8205(9)	4.1345(8)	**1.8273(1)**	2.1683(2)	2.3071(3)
bibtex	23.5730(3)	24.2592(4)	61.0501(8)	48.7797(6)	74.1217(9)	58.5996(7)	18.6720(2)	**16.4091(1)**	25.4803(5)
corel5k	107.0980(3)	120.1340(6)	111.2760(4)	199.0800(8)	320.5720(9)	120.5880(7)	96.3340(2)	**92.1240(1)**	114.5120(5)
yeast	6.5638(7)	7.1145(9)	6.4144(2)	6.4264(3)	6.9586(8)	6.5354(5)	**6.2443(1)**	6.5431(6)	6.4885(4)
Avg. rank	3.00	4.50	5.67	5.75	8.67	6.92	2.83	**2.75**	4.92

TABLE 15
The performance of the multi-label classification algorithms in terms of **ranking loss** ↓.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.0799(2)	0.0953(6)	0.0866(4)	0.0892(5)	0.2000(9)	0.1720(8)	0.0861(3)	**0.0780(1)**	0.1150(7)
Reuters(10)	0.0105(2)	0.0110(3)	0.0258(7)	0.0244(6)	0.0496(9)	0.0316(8)	0.0119(4)	**0.0084(1)**	0.0207(5)
Reuters(21)	0.0082(2)	0.0084(3)	0.0220(7)	0.0176(5)	0.0642(9)	0.0351(8)	0.0091(4)	**0.0069(1)**	0.0187(6)
Reuters(90)	0.0140(3)	0.0141(4)	0.0201(5)	0.0242(7)	0.1143(9)	0.0652(8)	0.0127(2)	**0.0095(1)**	0.0224(6)
medical	**0.0325(1)**	0.0351(3)	0.0540(6)	0.1029(7)	0.1030(8)	0.1135(9)	0.0344(2)	0.0366(4)	0.0507(5)
genbase	0.0045(4)	0.0046(5)	0.0060(7)	0.0037(2)	0.0059(6)	**0.0018(1)**	0.0061(8)	0.0077(9)	0.0043(3)
ohsumed_norm	0.0540(2)	**0.0507(1)**	0.1094(5)	0.1236(7)	0.2883(9)	0.2041(8)	0.1170(6)	0.0831(3)	0.1059(4)
emotions	0.1791(3)	0.1942(4)	0.2795(8)	0.2571(6)	0.3111(9)	0.2648(7)	**0.1711(1)**	0.1761(2)	0.2033(5)
tmc2007	0.0363(5)	0.0385(7)	0.0380(6)	0.0348(4)	0.1829(9)	0.0889(8)	**0.0208(1)**	0.0287(2)	0.0316(3)
bibtex	0.0781(3)	0.0792(4)	0.2399(7)	0.1961(6)	0.2972(9)	0.2556(8)	0.0620(2)	**0.0556(1)**	0.1085(5)
corel5k	0.1194(3)	0.1329(5)	0.1269(4)	0.2525(8)	0.5798(9)	0.1415(7)	0.1072(2)	**0.1009(1)**	0.1344(6)
yeast	0.1786(6)	0.2079(9)	0.1715(2)	0.1734(3)	0.2040(8)	0.1844(7)	**0.1706(1)**	0.1759(5)	0.1756(4)
Avg. rank	3.00	4.50	5.67	5.50	8.58	7.25	3.00	**2.58**	4.92

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TABLE 16
The performance of the multi-label classification algorithms in terms of **average precision** ↑.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.8561(3)	0.8346(7)	0.8492(5)	0.8563(2)	0.7626(8)	0.7548(9)	0.8525(4)	**0.8591(1)**	0.8354(6)
Reuters(10)	0.9680(2)	0.9675(3)	0.9412(7)	0.9441(6)	0.9365(8)	0.9230(9)	0.9630(4)	**0.9711(1)**	0.9578(5)
Reuters(21)	0.9576(2)	0.9569(3)	0.9274(7)	0.9358(6)	0.9076(8)	0.8716(9)	0.9472(4)	**0.9612(1)**	0.9447(5)
Reuters(90)	0.8899(3)	0.8912(2)	0.8770(6)	0.8748(7)	0.8284(8)	0.6579(9)	0.8783(4)	**0.8989(1)**	0.8780(5)
medical	**0.8719(1)**	0.8646(2)	0.7289(7)	0.6049(8)	0.7581(6)	0.4915(9)	0.8393(5)	0.8572(3)	0.8438(4)
genbase	**0.9945(1)**	0.9943(2)	0.9939(3)	0.9924(6)	0.9864(9)	0.9912(8)	0.9927(5)	0.9924(7)	0.9938(4)
ohsumed_norm	0.8052(2)	**0.8115(1)**	0.6667(5)	0.6421(6)	0.6258(7)	0.4998(9)	0.6238(8)	0.7097(4)	0.7419(3)
emotions	0.7857(2)	0.7660(5)	0.7088(9)	0.7314(6)	0.7092(8)	0.7208(7)	**0.7942(1)**	0.7832(3)	0.7688(4)
tmc2007	0.8498(4)	0.8451(5)	0.8277(7)	0.8380(6)	0.7244(8)	0.7215(9)	0.8782(2)	0.8642(3)	**0.8793(1)**
bibtex	0.5699(3)	0.5697(4)	0.3291(8)	0.3349(7)	0.4151(6)	0.2118(9)	0.5612(5)	0.5925(2)	**0.6072(1)**
corel5k	0.2890(3)	0.2762(5)	0.2770(4)	0.1624(8)	0.1479(9)	0.2163(7)	0.3089(2)	**0.3471(1)**	0.2647(6)
yeast	0.7526(5)	0.7022(9)	**0.7585(1)**	0.7570(2)	0.7344(8)	0.7488(7)	0.7544(3)	0.7524(6)	0.7542(4)
Avg. rank	**2.58**	4.00	5.75	5.83	7.75	8.42	3.92	2.75	4.00

TABLE 17
The performance of the multi-label classification algorithms in terms of **training time** (in hours) ↓.

	BR	CC	MLkNN	IBLR-ML	HOMER	PCT	RF-PCT	TSA	ML-Forest
scene	0.5468(6)	0.8686(7)	0.0313(4)	0.0175(3)	3.1079(9)	**0.0004(1)**	0.0106(2)	0.0402(5)	2.4658(8)
Reuters(10)	0.3058(6)	0.4330(7)	0.1532(4)	0.1890(5)	3.5188(9)	**0.0019(1)**	0.0333(2)	0.0357(3)	1.6901(8)
Reuters(21)	0.6063(6)	0.7417(7)	0.1867(5)	0.1740(4)	7.5049(9)	**0.0049(1)**	0.0857(3)	0.0551(2)	2.9268(8)
Reuters(90)	1.0601(6)	1.1896(7)	0.0830(4)	0.6115(5)	16.9018(9)	**0.0173(1)**	0.0813(3)	0.0447(2)	2.0974(8)
medical	0.0092(5)	0.0098(6)	0.0010(2)	0.0054(4)	0.1623(8)	**0.0004(1)**	0.0016(3)	0.0619(7)	0.2783(9)
genbase	0.0943(6)	0.0912(5)	0.0214(4)	0.0011(3)	0.8938(7)	**0.0001(1)**	0.0006(2)	12.2302(9)	4.8198(8)
ohsumed	7.6682(5)	8.1229(6)	0.9197(3)	0.7229(2)	42.5799(8)	**0.2098(1)**	5.5266(4)	99.3969(9)	33.4865(7)
emotions	0.3317(7)	0.4120(8)	0.0009(3)	0.0015(4)	3.4787(9)	**0.0001(1)**	0.0004(2)	0.2355(6)	0.1796(5)
tmc2007	1.3500(3)	1.4708(4)	2.6801(7)	1.8375(5)	13.5168(8)	**0.0139(1)**	0.1480(2)	2.4727(6)	166.0751(9)
bibtex	22.6319(6)	29.4215(7)	0.4625(2)	2.5392(5)	39.7952(8)	**0.0541(1)**	0.9940(3)	1.7673(4)	59.8075(9)
corel5k	6.7812(5)	8.9521(6)	0.1062(2)	11.1200(8)	39.4257(9)	**0.0715(1)**	0.7692(4)	0.4930(3)	9.7242(7)
yeast	7.1219(7)	7.1248(8)	0.0266(5)	0.0015(2)	0.0216(4)	**0.0005(1)**	0.0057(3)	0.0630(6)	7.3917(9)
Avg. rank	5.67	6.5	3.75	4.17	8.08	**1.00**	2.75	5.17	7.92

one algorithm rarely outperforms another algorithm on all criteria. With the results shown in Fig. 4 and Tables 13 to 16, we find that the performance of our proposed method is not significantly affected, it still can achieve competitive performance against TSA, BR, CC and RF-PCT in terms of the ranking-based metrics.

TABLE 18
The accuracy of ML-FOREST using different kernel functions against different number of trees K on the medical and scene data sets.

Data set	Kernel	1	5	10	20	50
medical	linear	0.696	0.728	0.734	0.743	0.751
	radial basis	0.719	0.719	0.728	0.737	0.742
	polynominal	0.729	0.732	0.738	0.744	0.747
scene	linear	0.608	0.705	0.717	0.718	0.719
	radial basis	0.585	0.714	0.718	0.718	0.718
	polynominal	0.609	0.612	0.710	0.715	0.719

4.2.6 PARAMETER SENSITIVITY AND TRAINING TIME

In this experiment, we investigate how different values of the number of tree K affect the classification performances of the proposed ML-FOREST algorithm. We vary the values of K from 1 to 50. The accuracy results of ML-FOREST using different kernel functions against different number of trees K on the medical and scene data sets can be found in Table 18. One observe that when the number of trees increases, the accuracy of the ML-FOREST algorithm increases. It has been shown that learning performance can be significantly enhanced when the ensemble has sufficient base learners. We try linear kernel, radial basis kernel, and polynomial kernel with parameters selected using cross-validation. We can see from Table 18 that the performance of ML-FOREST does not have significant difference while training with different kernel functions.

We also evaluate the robustness of ML-FOREST against the other two input parameters: the purity threshold λ and the SVM penalty C. The values 2^{-5}, 2^{-3}, ... 2 were considered for C and 0.4 to 0.9 for λ. The 3D graph in Fig. 5 shows how the accuracy of ML-FOREST varies against different values of λ and C. We can see that the accuracy of ML-FOREST increases when the value of λ increases and meanwhile the value of C decreases. We observe that the accuracy is degraded when λ is small. In this case, a number of irrelevant classes with confidences larger than λ are included within predicted labels, and thus the performance is degraded drastically. As a result, we randomly selected λ in the range $(0.9, 0.95)$ to set λ with a large value. The results on other data sets are similar. We also find that the performance of ML-FOREST will decrease if the penalty C is chosen too large or too small. In our experiment, we have used the value $C = 2^{-5}$ as default setting.

We also report the training times of each algorithm (see Table 17). The experiments are conducted on an Intel Xeon 2.4 GHz machine with 128 GB RAM running Windows Server 2012. The results show that PCT is the fastest method. The RF-PCT algorithm is implemented using multi-thread programming, and it ranks 2nd in terms of training time (the code is available at https://sites.google.com/site/qysite/).

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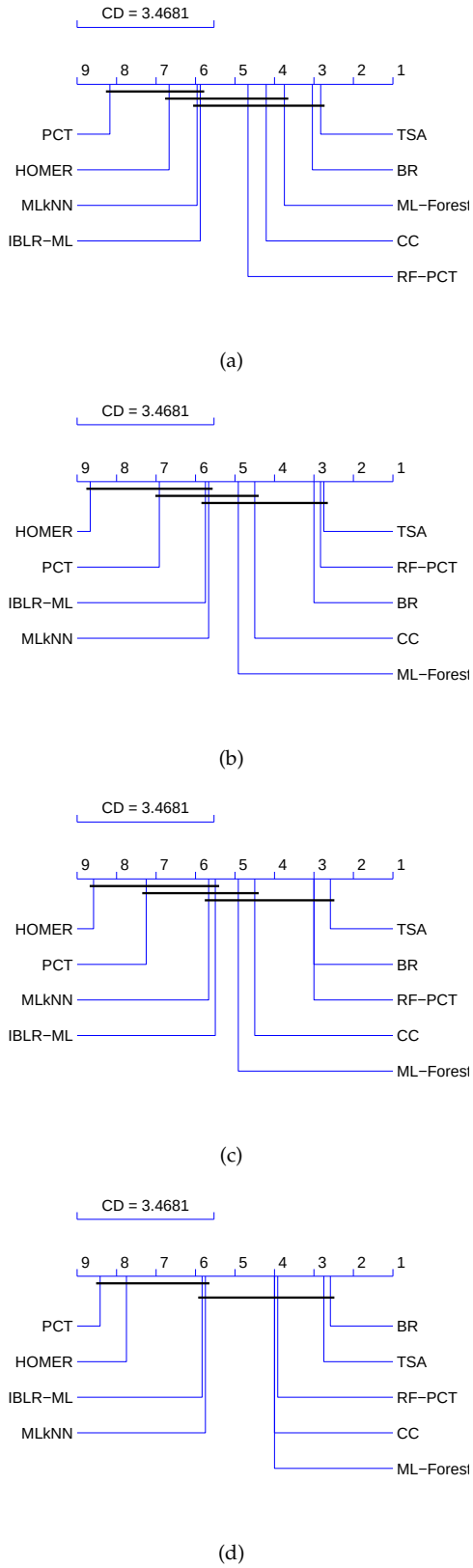

Fig. 4. The average ranks diagrams for the ranking-based measures: (a) one-error, (b) coverage, (c) ranking loss, and (d) average precision.

Tree based learning methods, such as ML-FOREST and HOMER, have the largest training time. HOMER is mainly due to the high computational cost in tuning the parameters in the model, while ML-FOREST is due to the high computational cost in learning multiple tree classifiers. The ML-FOREST algorithm is easy to be parallelized and thus the running time can be significantly improved in case using multi-thread parallel mechanism to implement it on computer with multi-core processors.

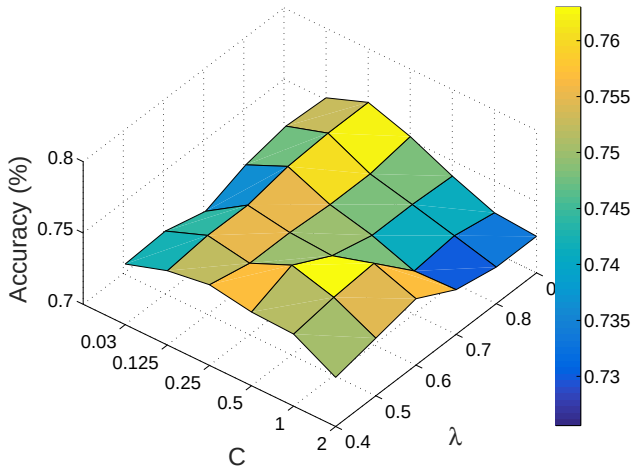

Fig. 5. The accurate of ML-FOREST with respects to the threshold λ and penalty C parameters.

5 CONCLUSION

In this paper, we have presented a new multi-label classification method, called ML-FOREST, to build an ensemble classifier. In ML-FOREST, we construct a set of hierarchical trees that is able to automatically exploit the label correlation, and develop a label transfer mechanism which identifies the relevant labels hierarchically.

ML-FOREST models the label dependency as a hierarchical scheme and performs the multi-label classification on this tree structure as a hierarchical decision process. As a result, ML-FOREST can have more discriminating ability than the first-order multi-label classification methods which only transform a multi-label problem into multiple separate and independent binary problems. Experimental results show that the proposed tree ensemble method is highly competitive to the state-of-the-art multi-label classification algorithms. Several works remain to be investigated in our future work:

1) ML-FOREST is a hierarchical tree ensemble algorithm to model the label dependency. Instead of using the linear SVM as base classifier in the hierarchy, utilizing some probabilistic base classifier (such as Bayesian approach) might fit more to estimate the conditional probability distribution $p(\mathbf{y}|\mathbf{x})$. This suggests one way to extend ML-FOREST.
2) Tsoumakas et al. [22] show that a hierarchical multi-label classifier model can be very efficiency on the tasks with a large number of labels if clustering technique is considered to organize the labels in growing the tree. It is interesting to apply Tsoumakas's idea to ML-FOREST approach.

3) In practice, the acquired multi-label data set can be imbalanced and noisy, and thus the hierarchical tree can be imbalanced, which degrades the generalization ability of the learning performance. Thus it is important to consider this challenging problem in the future.

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